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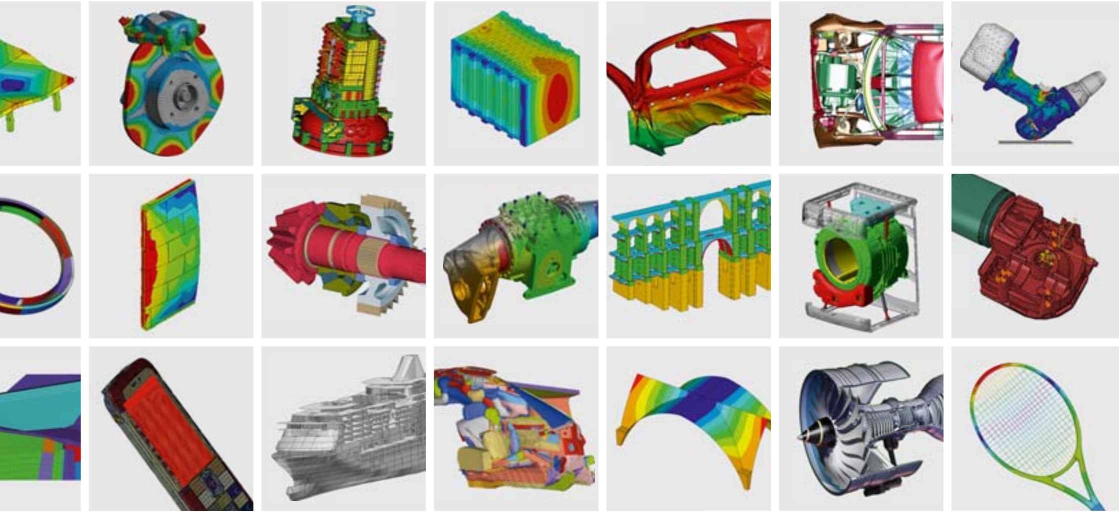

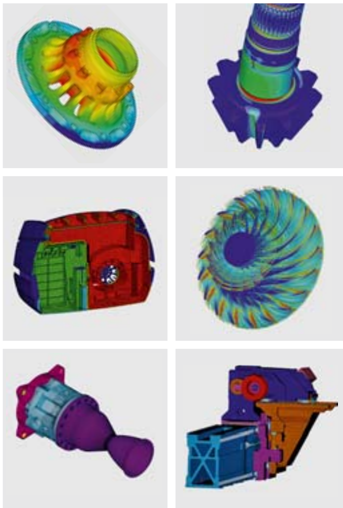

CAE-Software & Consulting

Robust Design Optimization (RDO) – Key technology for resource-efficient product development and performance enhancement

RDO-BOOKLET

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CONTENT

INTRODUCTION

CAE-based optimization looks back on a long tradition in engineering. The goal of optimization is often the reduction of material consumption while pushing the design performance to the boundaries of allowable stresses, deformations or other critical design responses. At the same time, safety margins should be reduced, products should remain cost effective and over-engineering should be avoided. Of course a product should perform effectively in the real world, with the variety of manufacturing, assembly and environmental conditions which may be expected to occur not only optimal under one possible set of parameter realizations. It also has to function with sufficient reliability under scattering environmental conditions. In the virtual world, the impact of such variations can be investigated through, for example, stochastic analyses leading to CAE-based robustness evaluation. If CAE-based optimization and robustness evaluation are combined, the area of Robust Design Optimization (RDO) is entered, which may also be called “Design for Six Sigma” (DFSS) or just “Robust Design” (RD).

The main idea behind such methodologies is the consideration of uncertainties within the design process. These uncertainties may have different sources: for example, variations in loading conditions, tolerances of the geometrical dimensions and material properties caused by production or deterioration. In the design optimization procedure, some of these uncertainties may have a significant impact on design performance and must therefore be considered.

Before entering the discussions of suitable and efficient algorithms for CAE-based RDO, some important considerations hould be pointed out when enhancing CAE-based optimization to deliver CAE-based RDO.

Garbage in – garbage out

In order to understand the influence of uncertainties, the best available knowledge about the expected uncertainties needs to be gathered gaining the best possible translation of this data into the statistical definition of the CAE-model. As an illustration, consider the conventional calculation of stress. It is clear that this requires a reliable value of Young’s modulus. The same question arises for stochastic analyses. If there are not trustable information on the essential input uncertainties, or no suitable approach to translate this information into statistical distributions of scattering parameters, a stochastic analysis should not be performed. In such a case this analysis would lead to useless estimates of the variations, sensitivities, etc.

While discussing how to formulate a suitable uncertainty model, one of the most important differences between CAE-based optimization and CAE-based robustness evaluation or even RDO should be noted. When simplifying an optimization task while using just a small subset of optimization parameters and a tiny parameter range you may not succeed in significantly improving the design. Nevertheless, any variation is a valid space of the optimization parameters and gives you valid information about the optimization potential corresponding to this space. There is no risk of obtaining inaccurate or dangerous predictions simply through the restrictions you have applied to the design space: at worst, you may miss solutions which lie outside that space. In other words, in a deterministic optimization the user can reduce almost arbitrarily large and complex parametric spaces to a handful of parameters with small ranges without the loss of confidence of the obtained optimization results.

In sharp contrast, the verification of product safety with a simplified robustness evaluation is only possible, if the unimportance of any neglected uncertain inputs is proven or their effect is covered sufficiently by safety factors. If significant important effects are neglected during the analysis, the robustness assessment based on this insufficient information may be much too optimistic and the results may assume an artificial safety. For this reason, it is recommended that any RDO task begins with a robustness evaluation of a represented design, introducing all possibly affecting uncertainties and making use of conservative estimates of their expected scatter. This enables us to determine which uncertainties are important and what accuracy of representation is necessary to introduce each uncertainty into the CAE model. Then it is secured that the uncertainty quantification and representation is appropriate, and proceed to answer the following questions:

- Is it sufficient to check the influence of our uncertain parameters by assuming a conservative upper and lower bound within a uniform distribution or is an more detailed identification of their statistical distribution type necessary?

- Is it sufficient to define single independent scattering parameters, should scalar parameters with pair-wise correlations be introduced, or is an utilization of more sophisticated spatial correlation models ("random fields") necessary to represent the properties of the uncertainties? (Wolff 2013, 2014)

Confident robustness measures need to be defined

By defining an RDO task, measures of variation will be included into the objectives and/or constraints. These statistical measures (mean value, standard deviation, higher order moments, safety margins or probabilities of exceeding a critical event) are outcomes of the stochastic analysis. Please note that all of these measures are estimates and their confidence has to be proven. In similarity to the verification of the mesh quality of a finite element analysis, the verification of each estimate of variance is necessary in order to be trusted in the predicted robustness of an investigated design. Everybody agrees that evaluating only 10 sample points will not lead to a confident assessment of a six sigma design. A six sigma design requires the proof that the probability of its failure is not larger than three out of a million realizations. 10 sample points are sufficient only to calculate a rough estimate of a mean value or a standard deviation. There will be no predictive value in using such an estimate to evaluate the small event probabilities associated with six sigma design analysis. A fundamental challenge of RDO is then how best to deploy your available solver time. Each design point will require many iterations to obtain accurate measures of robustness. However, if this means that too few design points are considered then the solution variation over the design space will be inadequately understood, and good solutions missed. Therefore, all RDO strategies need to estimate variation values with a minimal number of solver calls. To reach this goal, some methods make assumptions about the linearity of the problem or use response surface approximations spanning in the space of the scattering parameters. The accuracy of these assumptions or approximations must be demonstrated to satisfactorily prove the fulfilment of the targeted robustness and reliability requirements.

If there are only vague knowledge about the importance of the various uncertainties and their best available representation in a CAE model, a verification of the robustness of existing designs is strongly recommended before extrapolating robustness measures to future variants.

RDO is not just a small extension of an optimization problem

Often, in marketing or scientific publications, the RDO task is simplified by the assumption that the robustness space is a subspace spanned by the optimization parameters.

The suggested RDO strategies based on this simplification allow us to recycle solver runs from the optimization algorithms for the robustness evaluation and to reduce the additional effort of RDO compared to deterministic optimization. Unfortunately, for real world engineering applications there are additional uncertain parameters beyond those that drive the optimization. These might include loading conditions, material properties or environmental effects. A meaningful assessment of robustness will require the effect of these parameters to be included. As a consequence, it needs to be accepted that the optimization and robustness parameters span different domains, and it is therefore commonly found that solver runs in the optimization domain cannot be recycled directly to estimate robustness criteria and vice versa.

Therefore, it should be expected that substantial engineering robustness evaluations or RDO tasks always have to consider a significant amount of additional information for the input uncertainty, which will start with a large number of uncertain parameters and will need significant additional CPU requirements. In the light of these additional costs, it is all the more important to carefully plan a suitable algorithmic RDO workflow, double check any available uncertainty data and its best representation in the uncertainty model and carefully determine the most suitable measures of design robustness.

Consequently, it is recommended to start with an *iterative RDO* approach using decoupled optimization and robustness steps, including an initial sensitivity analysis in the domain of the optimization parameters as well as a subsequent sensitivity evaluation in the domain of uncertain parameters. This iterative approach helps to better understand the importance of each variable and the complexity of the RDO task in order to adjust the necessary safety margins. Only with this knowledge, and under circumstances where the iterative approach does not converge, should a *simultaneous RDO* task be defined. The iterative approach will be illustrated in our show case example.

Deterministic Optimization

In parametric optimization, the optimization parameters are systematically varied by mathematical algorithms in order to get an improvement of an existing design or to find a global optimum. The design parameters are defined by their lower and upper bounds or by several possible discrete values.

In real world industrial optimization problems, the number of design parameters can often be very large. Unfortunately, the efficiency of mathematical optimization algorithms decreases with an increasing number of design parameters. With the help of sensitivity analysis the designer can identify those parameters which contribute most to a possible improvement of the optimization goal. Based on this identification, the number of actual design parameters may be dramatically reduced and an efficient optimization can therefore be performed. This early sensitivity analysis may be useful in helping to decide whether the optimization problem has been formulated appropriately and whether the numerical CAE solver has behaved as expected.

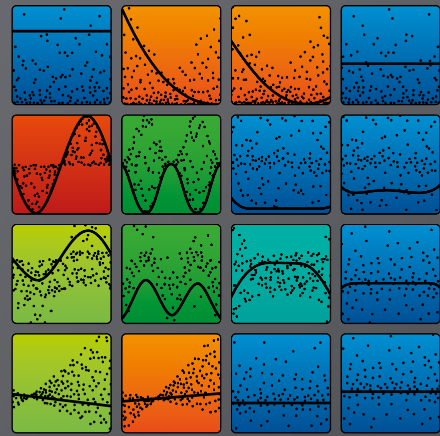
Robustness evaluation

As noted above, optimized designs will typically be sensitive to variation or scatter in their geometry, material parameters, boundary conditions and loads and so forth. A robust design process should predict how the optimized design is affected by this variation. Design robustness can be checked by applying stochastic analysis (such as Latin Hypercube Sampling) which is based on randomly generated collections of design variants possessing statistically scattered parameters and evaluating their performance. This will require the introduction of robustness measures such as mean value, standard deviation, safety margins to failure criteria as well as the probability of failure. If variation-based measures are chosen, the approach is called “variance-based robustness evaluation”. While using probability-based measures, such a procedure is called “probability-based robustness evaluation”, also known as “reliability analysis”.

Robust Design Optimization

In this booklet, different strategies to search for a robust design are presented and investigated with respect to their efficiency and applicability to time consuming numerical models. Iterative RDO has been described above, where deterministic optimization is combined with variance-based robustness analysis at certain points during the optimization process. In such a scheme, the frequency of coupling and interaction of both tasks has to be defined. During the deterministic cycles of the optimization, safety factors should also be defined to ensure that a sufficient distance to the failure criteria is maintained. These safety factors may be adjusted during the iterative RDO process. A final robustness and reliability proof will be mandatory at the end of the procedure. This procedure is state-of-the-art in the majority of publications on real world RDO projects (Roos and Hoffmann 2008), (Roos et al. 2009).

If the safety margins fluctuate within the optimization domain (for example due to the interaction of several failure phenomena) an iterative procedure may require a large number of iterations. In such a case, an automatic approach where the robustness criteria are estimated for every candidate in the optimization domain, a so-called nominal design, may be more efficient with respect to the CPU requirements. This procedure is called a simultaneous RDO approach. Since the robustness evaluation is performed as an internal loop within the global optimization loop, this approach is sometimes also called “loop in loop” RDO.



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GLOBAL SENSITIVITY ANALYSIS

By definition, sensitivity analysis considers how the variation in the output of a model can be apportioned, qualitatively and quantitatively, to different sources of variation of the input of a model (Saltelli et al. 2008). In order to quantify this contribution of each input parameter to each response variable under real world CAE conditions comprising non-linear, noisy problems with large number of parameters, variance based methods prove to be suitable. With these methods, the proportion of the output variance can directly be quantified, which is caused by each input variable variation. Such an approach can be applied equally to the domains of the optimization and the uncertainty parameters. In the optimization space, the continuous variables are represented with uniform distributions. In the uncertainty space, adequate distribution functions and correlation models for our parameters are utilized. Therefore, variance-based sensitivity analysis is suitable as preprocessing step for both an optimization and robustness, to investigate and identify variable importance as well as to estimate the amount of unexplained variation of response values which may occur from CAE solver noise, and to identify functional correlations to the input variation.

In contrast to local derivative-based sensitivity methods, global variance based approaches quantify the contribution with respect to the defined total variable variation. Unfortunately, sufficiently accurate variance-based methods require a huge numerical effort due to the large number of necessary simulation runs. Therefore

Figure 1: Flowchart of variance-based sensitivity analysis using the Metamodel of Optimal Prognosis

meta-models or simplified regressions are often used to represent the model responses by surrogate functions in terms of the model inputs. Many meta-model approaches are available and it is often not clear which one is most suitable for a given problem (Roos et al. 2007). Another disadvantage of meta-modeling is its limitation to a small number of input parameters. Due to the so-called “curse of dimensionality” there is a dramatic decrease in the quality of approximation for all meta-model types as the number of variables increases. As a result, an enormous number of samples are required to represent high-dimensional problems with sufficient accuracy. In order to overcome these problems, Dynardo developed the Metamodel of Optimal Prognosis (Most and Will 2008, 2011). In this approach, the optimal input parameter subspace together with the optimal meta-model type is automatically determined with the help of an objective and model-independent quality measure, the Coefficient of Prognosis (CoP).

In Figure 1, the general flow of this meta-model based sensitivity analysis is shown: after the definition of the design parameters and model responses the Design of Experiments or sampling methods generate a specific number of designs. These samples are evaluated by the solver and the model responses are determined for each design. Approximation models are built for each model response and assessed regarding their quality. Finally, variance based sensitivity indices are estimated by using the underlying approximation models.

The Design of Experiment (DoE) generates samples which are used to identify the parameter importance. Since one main part of the MOP/CoP algorithm is the reduction of the parameter space using only the important parameters, sampling strategies which vary every parameter in every design are needed. Please note that traditional deterministic DoE schemes like full factorial or D-optimal usually only vary on a few parameters each time. If the unimportant parameters are filtered out of the data, such schemes will lose the majority of design point information. Therefore, space filling, (quasi) random sampling methods seem to be more suitable for such tasks. In our experience, Latin Hypercube Sampling with reduced input correlation errors has been observed to be the most effective sampling scheme for the majority of investigated problems.

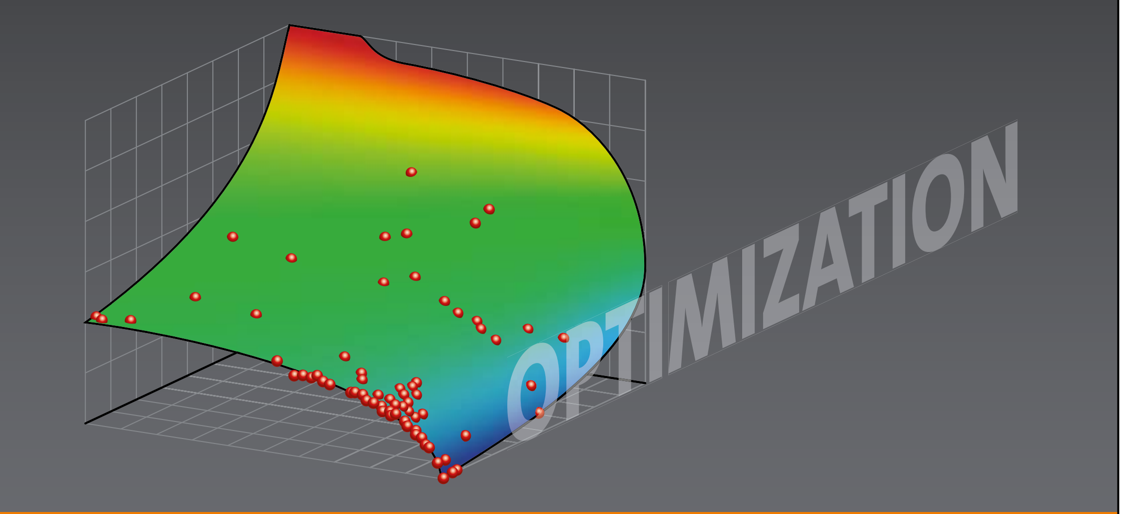
The assumption that non-trivial problems have a large number of potentially important input parameters, simple assumptions like pairwise linear correlations, as well as nonlinear dependencies of multiple input parameters including their

Simple Correlation Measures	Polynomial Regression Models	Metamodel of Optimal Prognosis
One-dimensional	Multi-dimensional	Multi-dimensional
Single parameter influence	All parameters or step-wise regression	Automatic identification of important parameters
Linear or monotonic dependencies	Linear or quadratic dependencies with interactions	Nonlinear (continuous) dependencies
No interactions	Linear interactions	Nonlinear interactions
No error measure	Goodness-of-fit or statistical tests as error measure	Prognosis quality as error measure (CoP)

Table 1: Comparison of global sensitivity methods to quantify the input parameter influence to the output variation: traditional one-dimensional correlation analysis, multi-dimensional polynomial regression and the Metamodel of Optimal Prognosis

interactions need to be investigated for a reliable assessment of the parameter importance. Simple linear correlation models and advanced nonlinear meta-models need to be evaluated in parallel. In order to identify the optimal meta-model including the best input parameter subspace, the model-independent quality measure Coefficient of Prognosis can be used. This measure estimates how well a meta-model can represent unknown data. Compared to classical measures, where the “goodness of fit” is only evaluated at the points which are used to build the model, the CoP gives much more reliable estimates especially for a large number of input parameters and a limited number of available designs.

Finally, the automatic MOP/CoP approach solves three very important tasks of a parameter sensitivity analysis: the identification of the most important combination of input parameters together with the best suitable surrogate function in order to obtain the optimal forecast quality of the response variation. This algorithm gives the main value to our customers. By “understanding the design”, the important input variations are detected regarding the response variation. Further, information about the extent of non-linearity of the problem is derived. This is combined with best result extraction to obtain response values which have the highest possible forecast quality, representing the response variation by the input parameters, and are minimally effected by solver and extraction errors. In Table 1, the significant improvements with respect to simple correlations and multi-dimensional polynomials are summarized.



MULTIDISCIPLINARY DETERMINISTIC OPTIMIZATION

Besides using the sensitivity analysis to reduce the number of optimization parameters to the most important ones, it can also be used to learn more about the character of an optimization problem:

- How non-linear and noisy are the response functions?
- Is an improvement of the result extraction of the solver output possible in order to achieve a better representation of the responses to the optimization parameters?
- Is an adjustment of the variation range of the optimization parameters necessary?
- How many responses are in conflict with each other?
- Has the initial scan of the design space already identified good designs which could be introduced as starting points for the following optimization algorithm?

Consequently, the findings from the sensitivity study will result in a qualified definition of the input parameter space, its objectives and constraints. It may even assist in making an appropriate choice of optimization algorithm and the corresponding settings.

In Figure 2, the recommended workflow of a single-objective optimization procedure is shown. After the definition of the design space, a sensitivity analysis is performed. As a result of the sensitivity analysis, unimportant parameters are filtered and an optimal meta-model is obtained for each response. This is followed by the definition of objectives and constraints. In a first step, the MOP approximation can be used to run a pre-optimization without expensive solver calls. Since the approximation models may locally differ from the solver solution, the obtained optimal design should be verified with an additional CAE-run. If the obtained design shows a sufficient improvement, the analysis may be stopped here. Otherwise, further improvements of the design are possible by adapting the approximation functions with additional designs or by running a direct search.

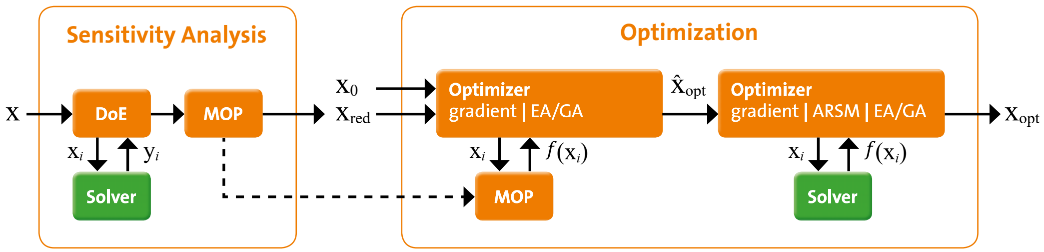


Figure 2: Recommended workflow for a single-objective optimization: from a full parameter set X the sensitivity analysis identifies the important parameters $\mathbf{X}_{red}$; together with the start design $\mathbf{X}_0$ the optimization is performed and an optimal design $\mathbf{X}_{opt}$ is found.

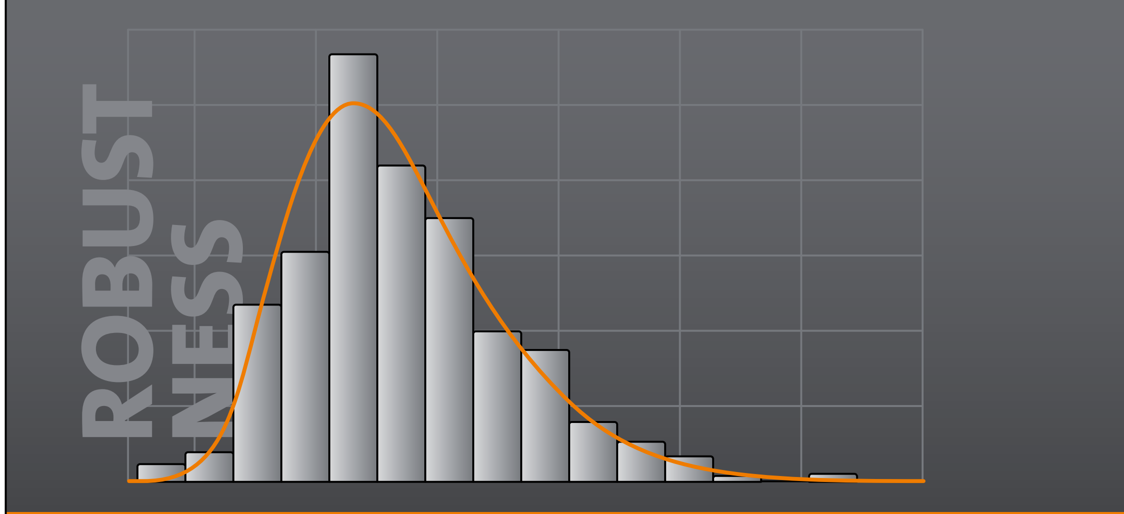
For the search of an optimal design on an approximation function or even with direct solver runs, a huge number of optimization algorithms can be found in literature. They can be classified in gradient-based algorithms like steepest descent and Newton methods (Kelley 1999), heuristic gradient-free approaches like grid or pattern search, adaptive response surfaces and simplex optimizer, and also nature-inspired search methods like genetic and evolutionary algorithms (Bäck 1996), particle swarm optimization (Engelbrecht 2005) and simulated annealing.

In Table 2 (see next page), a set of representative methods is given and assessed with respect to its field of applications.

Often, several objectives have to be addressed in the optimization task. In cases where these objectives are in conflict with each other and a suitable weighting of these objectives is not possible with the available knowledge, the application of single-objective optimization methods may be difficult. In such cases, the optimization can be performed by searching simultaneously for different possible compromises between the conflicting objectives, a so-called Pareto frontier. Once this frontier is obtained, more qualified decisions are possible in order to specify further requirements for the selection of a suitable design out of this frontier or its further improvement by single-objective methods. Today, genetic and evolutionary algorithms as well as particle swarm optimization methods are often the methods of choice for an efficient Pareto search.

Algorithm	Global search	Local search	No. of parameter	Constraints	Failed designs	Solver noise	Discrete parameters
Gradient based (NLPQL/ SQP)	no	ideal	<= 20	many	no	no	no
Downhill simplex method	yes	ideal	<= 5	few	yes	minor noise	ordered discrete
Response surface methods	ideal	ideal	<= 20	many	yes	yes	ordered discrete
Evolutionary & genetic algorithms	ideal	yes	many	yes	many	yes	yes
Particle swarm optimization	ideal	yes	many	few	yes	yes	ordered discrete
Simulated annealing	yes	no	<= 20	few	yes	yes	yes

Table 2: Comparison of common optimization methods: the maximum number of parameters is defined regarding the efficiency of the optimization method compared to the other algorithms



ROBUSTNESS EVALUATION

As noted, satisfying design requirements will necessitate ensuring that the scatter of all important responses by fluctuating geometrical, material or environmental variability lies within acceptable design limits. With the help of the robustness analysis, this scatter can be estimated. Within this framework, the scatter of a response may be described by its mean value and standard deviation or its safety margin with respect to a specified failure limit. The safety margin can be variance-based (specifying a margin between failure and the mean value) or probability-based (using the probability that the failure limit is exceeded). In Figure 3, this is shown in principle.

In the variance-based approach, the safety margin is often given in terms of the according standard deviation of the corresponding response. A “six sigma” design should fulfil a safety margin of six times the standard deviation. Assuming a normally distributed response, the classical six sigma concept considers an additional safety margin of 1.5 times the standard deviation. The 4.5 sigma margin of a normal

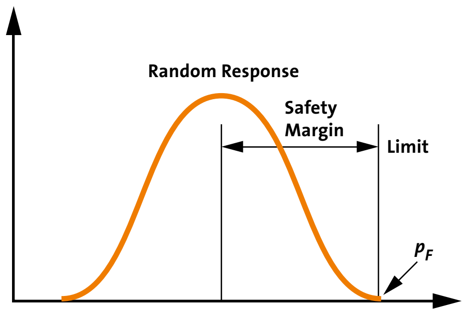


Figure 3: Scatter of a fluctuating response with safety margin (distance between mean and the failure limit) and the corresponding probability of failure p_F

distribution corresponds to a failure rate of 3.4 defects out of one million design realizations. The assumption of a normally distributed response may not be valid if non-linear effects dominate the mechanisms of failure (Most and Will 2012). In such cases, the extrapolation of rare event probabilities, like 3.4 out of a million, just from the estimated mean value and standard deviation may be strongly erroneous. Thus, the assumption of a normal distribution should be verified at least at the final RDO design. If this is not the case, the probability of failure should be estimated with the more qualified reliability analysis.

Definition of uncertainties

In variance-based and probability-based robustness evaluation, the definition of the uncertainty of scattering input variables plays an important role. A wrong assumption in this definition may cause large errors in the estimated robustness and safety measures. This may lead to useless or even dangerous results. Therefore, this step should be carefully validated. If only insufficient data are available, a conservative assumption of lower and upper bounds using a uniform distribution is recommended.

The scattering inputs can be described as scalar random variables or as stochastic processes and fields. Scalar random variables represent the scatter of a single variable independently by a probability distribution. With the help of statistical moments such as mean value and standard deviation and a specific distribution type like normal, log-normal or uniform, the scatter can be represented in mathematical form. In addition, discrete random variables can be modeled – for example by a binomial distribution.

If the random variables are assumed to be independent but they are not in reality, a combination of physically non observable extreme values may be possible. As an example: if the yield stress and the tensile strength of steel are defined as independent scattering variables, some realizations will lead to lower tensile strength than yield stress, which is obviously nonphysical. Therefore, material parameters or loading conditions which show significant dependencies need to be considered in the uncertainty model. For normally-distributed variables, this dependence can only be linear and can be represented in closed form. However, industrial applications require the adequate representation of dependencies between different distribution types. This can be realized by sophisticated correlation models like the Nataf approach (Nataf 1962).

If the spatial distribution of geometrical and material properties may affect the physical behavior of the design, a more detailed representation of the spatially correlated scatter is necessary. With the help of the random field methodology, complex spatial distributions and correlations can be analyzed and reconstructed for a subsequent robustness evaluation. The continuous field is discretized in this framework by a reasonable number of scalar random variables which are associated with scatter shapes. Its statistical evaluation is then straightforward (Wolff 2013). The random field concept can be used to model spatially distributed input variables as well as to analyze spatial response values. For example, random fields may help to detect hot spots in the response values which are responsible for local failure mechanisms.

In case of only roughly known input scatter, a robust design optimization may be used to estimate the maximum possible scatter for specific input parameters. In such procedure, a qualified knowledge of the input uncertainty is not necessary (Most & Will 2017).

Variance-based robustness analysis

Today, the majority of RDO approaches uses variance-based robustness measures in order to minimize the variation of a response with or without constraining the mean, which is known as the Taguchi approach, or to reach a certain level of safety quantified by the safety margin or sigma levels, where Design for Six Sigma is one possible concept.

Since the Taguchi approach is well-known in the industrial Six Sigma community, the application of this strategy to the virtual product development often needs some extensions. The main aim of the Taguchi approach is to reduce the scatter of a final product parameter to an acceptable level. Within a production line, the exceeding of this limiting level can usually be measured with a high accuracy. In this case, a sensitivity analysis is performed in order to detect the responsible scattering input variables. With this information, some of the important input sources may be reduced in order to result in smaller product scatter. But the reduction of the input scatter is only one possibility route to reduce the output scatter. This approach inevitably drives the process to increasingly tight tolerances with respect to the materials and the production process. This might extremely increase the production costs. In virtual prototyping, by contrast, designs with inherently reduced sensitivity are usually preferred to input scatter. Often, Taguchi based RDO strategies try to reduce the sensitivity to the input scatter only and aim for designs with the lowest standard deviation. But a design with a very low standard deviation can still violate the safety constraints. On the other hand, the Taguchi approach may result in designs which are over engineered and not cost-effective. Therefore, it is recommended in the framework of variance-based RDO that the target is not simply the lowest possible standard deviation, but also to consider a sufficient safety margin to failure or operation limits. As a consequence, the proof of a sufficient safety margin to failure and operational limits should be the primary goal of RDO in virtual prototyping. In case of stringent requirements with respect to design tolerances (as in Design for Six Sigma), a final reliability proof should be considered.

Besides these fundamental questions, approaches for variance-based robustness evaluations need to estimate the necessary statistical measures as mean value

Approach	Degree of non-linearity	Distribution types	No. of parameters
First Order (linearization)	linear	only normal	<= 50
Polynomial chaos expansion	linear & quadratic	only few	<= 10
Global Response Surface	linear & quadratic	arbitrary	<= 10
Monte Carlo methods (LHS)	arbitrary	arbitrary	many

Table 3: Comparison of statistical methods for variance-based robustness evaluation

and standard deviation with a sufficient confidence. Based on the initial uncertainty definition, several approaches for their determination are available. An overview is given in Table 3 (see next page).

In case of a linear dependence between input variables and responses, the mean value and the standard deviation can be calculated analytically in a closed form. Therefore, some methods use a linearization around the mean for these estimates (e.g. First Order Second Moments), which is very efficient for a small number of input parameters. However, in case of non-linear dependencies, such a procedure may obtain strongly erroneous statistical estimates. Similar approaches by using a global linear or quadratic response surface have the same limitations: if the assumed linear or quadratic dependence is not valid, the estimated safety level may be far away from the real value. This is even the case for the polynomial chaos expansion approach: if the expansion can represent the physical behavior with low order basis terms, the safety assessment is confident and can be achieved efficiently. If this is not the case, the results of the low order approximations are not suitable for a safety assessment and higher order terms are needed which dramatically increases the number of necessary solver runs.

For industrial applications with a larger number of scattering inputs and non-linear dependencies, Monte Carlo based methods are more suitable (Will 2007). The Latin Hypercube Sampling (LHS) is an approach where the distribution of the samples is optimized with respect to small errors in the statistical estimates. This method does not assume any degree of model behavior and can also handle discontinuous responses. Furthermore, it works independently of the number of input parameters.

Rough estimates of mean and standard deviation are possible with just 20 solver runs. More precise estimates of mean and standard deviation can be obtained by using 50 to 100 samples. Based on the evaluated data and the estimated scatter of the responses, variance-based sensitivity measures can be evaluated in order to further analyze the source of uncertainty. From our experience, using a small LHS sample set to estimate standard deviation is an effective method which is also robust against system nonlinearity. By fitting the distribution function into the histogram of the response, the window of probability based on standard deviation as well as on fitted distribution functions can also be verified.

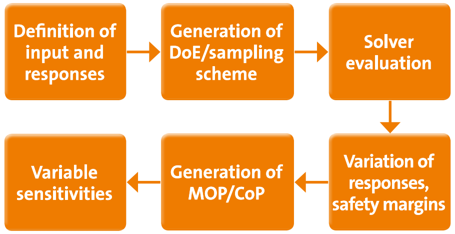


Figure 4: Flowchart of the variance-based robustness evaluation with an included sensitivity analysis

In Figure 4, the overall workflow of the variance-based robustness analysis is illustrated. Based on the initial definition of the random parameters, representative samples are generated and evaluated by the solver. The solver results are used to estimate the statistical properties and to perform a sensitivity analysis with help of the MOP approach. If the design does not fulfill the robustness requirements, the MOP/CoP analysis helps to identify these input parameters which are responsible for the violation of limits.

Reliability-based robustness analysis

Variance-based measures are often used within the RDO workflows due to their efficiency with respect to the number of solver runs. Therefore, it is very important that, at least for the final design, the targeted probability of exceeding a failure limit is verified. In engineering applications, reliability levels of at least 3 sigma (1.3 out of 1000) are usually required for non-critical products like high end consumer goods, while up to 5 sigma (less than one failure in one million designs) is required for safety relevant critical components. These kinds of rare events are usually connected to the non-linearity of the design being considered. Thus, it becomes difficult to estimate the probability distribution of relevant response values (e.g. maximum stresses). This is also true with respect to rare events where sufficient accuracy can only be reached if variance based methods or low-order approximations based on linearization or series expansions are applied. Therefore, the estimates of small probabilities have to be verified with a qualified reliability analysis.

With the reliability method, the probability of reaching a failure limit is obtained by an integration of the probability density of the uncertainties in the failure domain as shown in Figure 5 (see next page). One well-known method is the Monte Carlo Simulation, which can be applied independently of the model non-linearity and the number of input parameters (Rubinstein 1981). This method is very robust and can detect several failure regions with highly non-linear dependencies. Unfortunately, it requires an extremely large number of solver runs to prove rare events.

Therefore, more advanced sampling strategies have been developed like Directional

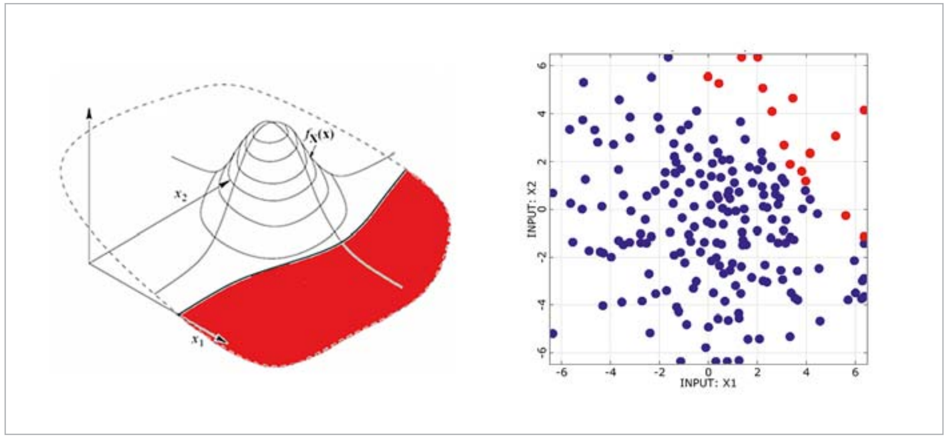


Figure 5: Reliability analysis as multi-dimensional integration of the probability density of the uncertainties of the inputs over the failure domain (left) and integration by Monte Carlo Simulation (right)

Sampling, where the domain of input variables is scanned by a line search in different directions, or Importance Sampling, where the sampling density is adapted in order to cover the failure domain sufficiently and to obtain more accurate probability estimates with much less solver calls. Other methods like the First or Second Order Reliability Method (FORM & SORM) are still more efficient than the sampling methods by approximating the boundary between the safe and the failure domain, the so-called limit state. In contrast to a global low order approximation of the whole response, the approximation of the limit state around the most probable failure point (MPP) is much more accurate. Classically, only one dominant failure point could be found and evaluated. This limitation holded even for the Importance Sampling Procedure Using Design points (ISPUD), where the non-linearity of the limit state can be considered by a sampling around the MPP. A good overview of these “classical” methods is given in Bucher (2009). Recently, these methods have been extended for multiple failure regions (Rasch et al. 2019).

For a successful application of global response surface methods, it is necessary to assure that the region around the most probable failure point is approximated with sufficient accuracy. This can be reached by an iterative adaptation scheme, where new support points are generated in this region. With this improvement, two or three important failure regions can also be represented with a small number of solver runs (Roos & Adam 2006).

In reliability analysis, where small event probabilities have to be estimated, one has to pay special attention at an acceptable level of confidence obtained by the algorithms in order to detect the important

Approach	Non-linearity	Failure domains	No. parameters	No. solver runs
Monte Carlo Simulation	arbitrary	arbitrary	many	>10^4 (3 sigma) >10^7 (5 sigma)
Directional Sampling	arbitrary	arbitrary	<= 10	1000-5000
Adaptive Importance Sampling	arbitrary	one dominant	<= 20	1000-5000
FORM, SORM, ISPUD	continuous	few dominant	<=20	200-500
Adaptive Response Surface Method	continuous	few dominant	<=20	200-500

Table 4: Comparison of different qualified reliability methods

regions of failure. Otherwise, they may estimate a much smaller failure probability and the safety assessment will be much too optimistic.

The available methods for an efficient reliability analysis try to determine where the dominant failure regions are and to concentrate the simulation effort in those regions in order to drastically reduce the necessary CAE simulations. This is necessary to become candidates of reliability for real world applications. Of course, there is always a risk that experimenting with such approaches will lead to inappropriate short cuts, perhaps missing the failure domain and providing too optimistic an estimation of failure probability. Therefore, it is strongly recommended that at least two different reliability methods are used to verify variance-based estimates of the failure probability in order to make reasonable design decisions based on CAE-models.

ROBUST DESIGN OPTIMIZATION

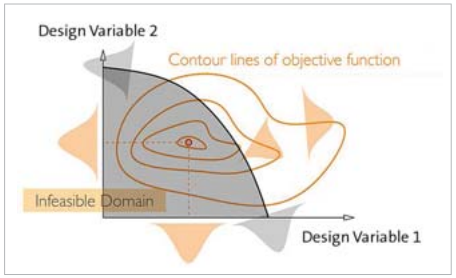


Figure 6: Robust Design Optimization – optimization considering uncertain input parameters and responses as well as objective and constraint functions

In the framework of Robust Design Optimization, the deterministic optimization methods are now extended by considering uncertainties of specific input variables. With help of a statistical evaluation of the no longer deterministic objective function and constraint conditions, the design is driven to a region where the robustness requirements are fulfilled while the desired performance is optimal.

The mathematically most-accurate way to obtain a robust design which fulfills even the requirements of a small failure probability would be to couple a suitable deterministic optimizer with a reliability analysis and to formulate the constraint conditions with respect to the required safety level. This approach is called reliability-based Robust Design Optimization and may be suitable for fast-running simulation models (Most & Neubert 2013). Due to the high numerical effort of evaluating each and every design point with an additional reliability analysis – indeed, repeating this with a second algorithm - for most real world applications it is not possible

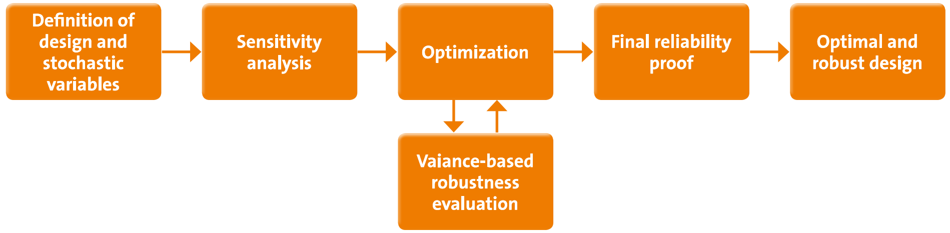


Figure 7: Flowchart of simultaneous variance-based Robust Design Optimization approach with final reliability proof

to perform such a sophisticated analysis. However, it is not necessary to prove the safety level of each design during the optimization process with such a high degree of confidence. Therefore, simplified methods have been developed. A first step to improve the efficiency and the robustness of a simultaneous Robust Design Optimization is to estimate the safety level with variance-based robustness measures. Taking into account an additional safety margin for approaches such as Design for Six Sigma, the coupled analysis can be performed by using only 20 to 50 samples for each nominal design in order to get reasonable estimates and to drive the optimizer in the right direction. Nevertheless, a final reliability proof should be considered again verifying the rough assumptions in the variance-based approach. In Figure 7 such a workflow is illustrated.

One important advantage of this approach is that the expensive reliability analysis only has to be performed at the end of the procedure. Furthermore, due to the dimension-independent estimates of the variance based measures, a large number of uncertain inputs can be considered. However, the numerical effort is still 20 to 50 times larger than the deterministic optimization with safety factors. This approach is thus limited to computational less expensive simulation models.

In Most & Will 2017, the simultaneous variance-based RDO approach was used to estimate the possible scatter of the input variables with respect to a given output variation. In this special case, the scatter of selected geometry properties itself were treated as optimization parameters. By using a multi-objective optimization procedure, the trade-off between maximum allowed input scatter and invest of material could be illustrated depending on the required safety level.

To overcome the limitation, global response surfaces are often applied. In case of simultaneous Robust Design Optimization, the approximation function has to consider all design parameters and all of the uncertain inputs. For cases where the number of uncertain inputs is large, an efficient application of such an approach is not possible (Most & Will 2012), since the number of necessary support points increases dramatically. However, a reduction in the number of inputs is dangerous

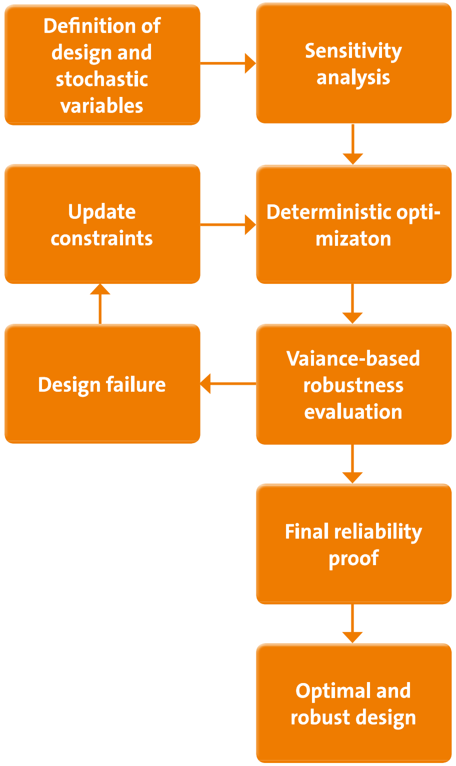


Figure 8: Flowchart of an iterative Robust Design Optimization with final reliability proof

since their influence may change when moving the nominal design through the design space. However, in some cases, this approach may perform well, but a final robustness evaluation with direct solver calls should always be performed in order to verify the approximated safety level.

In our experience, an iterative Robust Design Optimization procedure is usually more applicable to industrial applications than the simultaneous approach. In this procedure, the deterministic optimization has to consider safety factors to assure a certain safety margin of the critical responses. After a first optimization step, the robustness measures are evaluated and the safety level is assessed. These safety factors should be chosen such that the robustness requirements are fulfilled. Generally, the safety factors are not known a priori. In this case, a suitable initial guess is specified and the initial deterministic optimization is performed. If the safety requirements are not fulfilled, the responsible safety factors have to be increased and the deterministic optimization has to be repeated. With this approach, a robust design is usually found within 3 to 4 iterations. If the robustness criteria are expected to be fulfilled, a reliability proof is again necessary to verify higher safety levels. In Figure 8, the iterative approach is shown.

In Most et al. 2017, an iterative reliability-based Robust Design Optimization is presented. In this study a variance-based robustness approach was not applicable due to a discrete limit state function, which did not allow to estimate safety factors. Nevertheless, the iterative approach could reduce the numerical effort dramatically compared to the simultaneous Robust Design Optimization.

ILLUSTRATIVE EXAMPLE: ROBUST DESIGN OF A STEEL HOOK

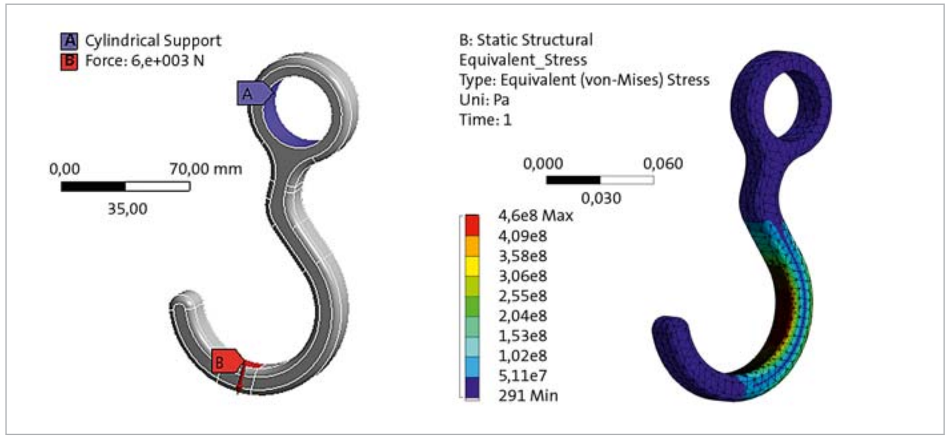
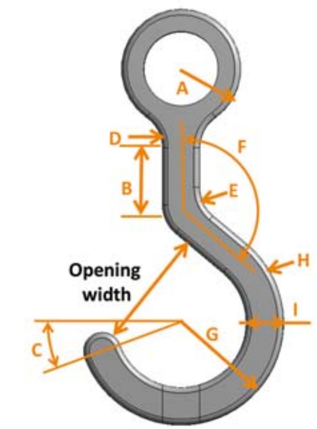


Figure 9: Initial geometry, loading and boundary conditions and finite elements representation

In this example, a steel hook subjected to a vertical load of 6000 N is investigated. The hook is analyzed by linear finite elements and has a cylindrical support at its head which can perform free rotations. The numerical simulation, including geometry modeling and automatic meshing, is done using ANSYS Workbench. The Robust Design



A	Outer Diameter	28-35 mm
B	Connecting Length	20-50 mm
C	Opening Angle	10-30°
D	Upper Blend-Radius	18-22 mm
E	Lower Blend-Radius	18-22 mm
F	Connecting Angle	120-150°
G	Lower Radius	45-55 mm
H	Fillet Radius	2-4 mm
I	Thickness	15-25 mm
	Depth	15-25 mm

Figure 10: Geometry parameters considered in the optimization with corresponding bounds

Optimization task in this example is to minimize the mass while the maximum stress should not exceed a failure limit of 300 MPa. The required safety level is defined with a 4.5 sigma safety margin corresponding to a failure rate of 3.4 out of a million.

For the optimization, 10 geometric properties are considered as design parameters. They are illustrated with their ranges in Figure 10. As a deterministic constraint, the opening width should not exceed 50 mm. These 10 geometric parameters are also considered as uncertain parameters for the robustness evaluation. Additionally, the scatter of the material parameters and the force value in tangential direction are taken into account. In order to consider non-tangential loading conditions, additional zero mean force components are assumed for the other directions. Table 5 gives an overview of the assumed distributions and variations of these uncertain inputs. Since the design parameters are changed during the optimization process, the mean values of the corresponding random variables have to be adjusted for the robustness evaluation.

Robustness analysis of the initial design

In the first step, a robustness evaluation of the initial design is performed. For this purpose, the geometry, material parameters and the force values are defined with the distribution and the variation given in Table 5. With this definition of the input scatter, a Latin Hypercube Sampling using 100 samples is generated and evaluated by the solver. In Figure 11, the observed scatter of the maximum stress value including its statistical properties is given. The figure shows, that the safety margin between the mean and the limit is only 0.6 sigma, which is much less than the required 4.5 sigma. The corresponding probability of failure is 0.24. Based on these results, the user can judge that the optimized design is not robust. Applying the MOP approach to distinguish between the input uncertainties,

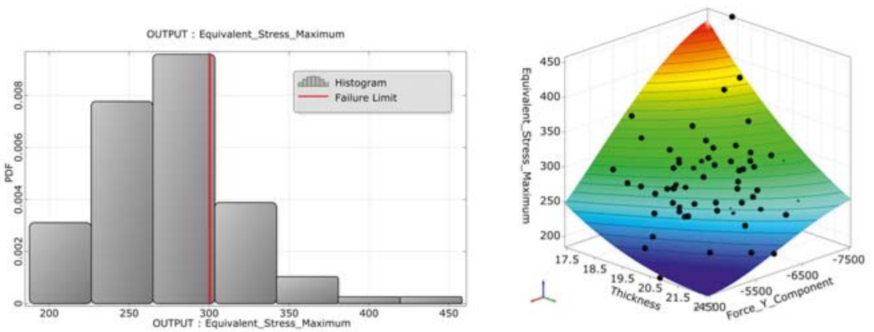
	Design parameter	Distribution	Mean value	Standard deviation
Outer diameter	yes	normal	(32 mm)	1 mm
Connection length	yes	normal	(40 mm)	1 mm
Opening angle	yes	normal	(20°)	2°
Upper blend radius	yes	normal	(20 mm)	1 mm
Lower blend radius	yes	normal	(20 mm)	1 mm
Connection angle	yes	normal	(130°)	2°
Lower radius	yes	normal	(55 mm)	1 mm
Fillet radius	yes	normal	(3 mm)	0.2 mm
Thickness	yes	normal	(20 mm)	1 mm
Depth	yes	normal	(20 mm)	1 mm
Young's modulus	no	log-normal	2e11 N/m²	1e10 N/m²
Poisson's ratio	no	log-normal	0.3	0.015
Density	no	log-normal	7850 kg/m³	157 kg/m³
Force x-direction	no	normal	0 N	100 N
Force y-direction	no	normal	6000 N	600 N
Force z-direction	no	normal	0 N	100 N

Table 5: Comparison of different qualified reliability methods

the load can be discovered as the critical influence. A limitation of the scatter of this boundary condition does not seem possible since it would limit the applicability of the product. Therefore, in the next step, a modification of structure is applied in order to be more robust against the defined input scatter level, which results in an increase of the design "in-built" safety margin. From the observed scatter of the initial design, it can be estimated that a mean value of maximum stress with 180 MPa would lead to the required safety margin of 4.5 sigma. Therefore, the next step is the performance of a deterministic optimization with reduced deterministic stress constraint. This is the first adaptation step in the iterative Robust Design Optimization.

Sensivity analysis with respect to the design parameters

In the following step, a sensitivity analysis should identify the most important design parameters. For this purpose, a Latin



Statistical data			
Min:	187.278	Max:	457.646
Mean value:	274.063	Standard deviation:	43.5748
CoV:	0.158996		
Skewness:	0.965973	Excess kurtosis:	2.32171
Limit : Failure Limit			
	Lower value = not set	Upper value = 300	Total
P_rel:		0.24	0.24
Sigma-Level:		0.59524	

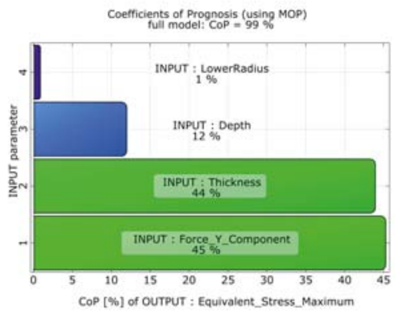


Figure 11: Results of the robustness evaluation of the initial design

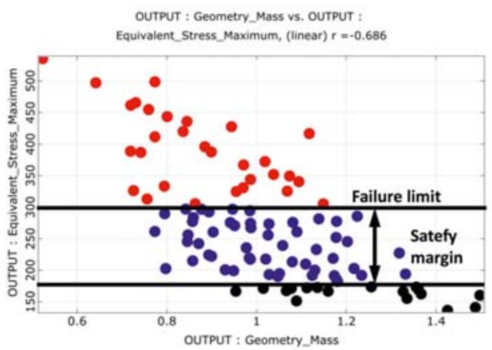


Figure 12: Mass and maximum stress as conflicting responses; the failure limit and an assumed safety margin corresponding to a constraint of 180 MPa

Hypercube Sampling with 100 samples is generated within the bounds of the 10 design variables and each design is evaluated by the simulation model. The mass, the maximum stress and the opening width are evaluated as model responses.

Figure 12 depicts the obtained mass and stress values. The figure indicates that by decreasing the mass, the maximum stress increases. 30% of the designs exceed the stress failure limit. Furthermore, only 17% of the designs fulfill the stress constraint

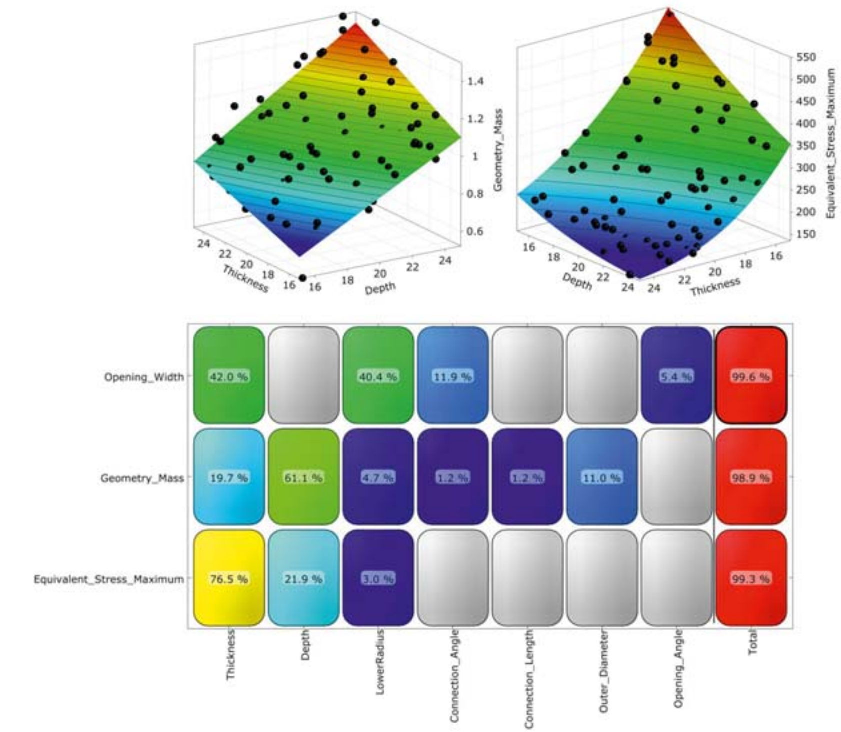


Figure 13: Results of the sensitivity analysis: 6 design parameters mainly influence the mass; the maximum stress can be explained with only 3 design parameters

condition. Applying the MOP approach, an almost perfect representation of the response values can be obtained with only some of the design parameters beeing relevant. The lower radius, the depth and thickness are important for mass and maximum stress as shown in Figure 13. The connection length and the outer diameter are only important for the mass. For each of the latter parameters, a positive correlation with the mass can be observed as illustrated in Figure 14 (see next page). Therefore, the minimum mass can be obtained using the lower bounds for these two parameters. As an outcome of the sensitivity analysis, these decisions have been made for the following optimization steps:

- The three blend radii are not important for the responses and are taken with their reference values.
- The connection length and the outer diameter are only important for the mass and are considered with their minimum values.

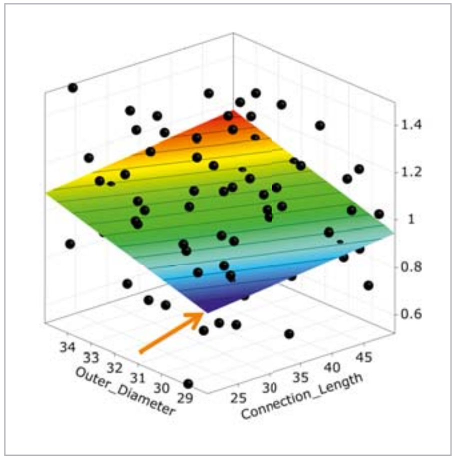


Figure 14: Influence of the connection length and the outer diameter to the mass

- Only the thickness, the depth, the lower radius, the outer diameter and the opening angle have to be considered as design parameters during the optimization. Their influence to the mass and maximum stress is contrary, thus an optimal combination has to be found during the optimization.

Deterministic optimization

In the first optimization step, the minimization of the mass is considered as an objective function. As a constraint condition, the maximum stress is limited to 180 MPa assuming a safety factor of about 1.7 to the stress limit of 300 MPa. With help of the already available MOP a pre-optimization can be done without any solver calls. As an optimizer on the MOP approximation functions, a gradient based approach is used which could improve the design from initially 1100 g to 840 g. However, due to local approximation errors, the obtained optimum is valid on the MOP approximation, but the verification with a direct solver call shows a slight violation of the constraint conditions. Nevertheless, this design is used as an appropriate start design for a direct optimization since it almost fulfills the constraint condition. As a direct optimizer, the gradient free Adaptive Response Surface Method is chosen. This algorithm converges quite fast if the responses show a global trend. Furthermore, it can handle solver noise, which might occur in this example in the stress values due to the automatic meshing procedure. The ARSM optimizer converges for this example within 10 iterations using 150 designs. The obtained optimum has a mass of 854 g while both constraint conditions are fulfilled. In Figure 15, the initial and the optimized design are compared.

Robustness evaluation

For the optimized design, a robustness evaluation is performed using 100 Latin Hypercube samples. The resulting scatter of the maximum stress is shown in Figure 16. As indicated, now the safety margin is about 4.7 sigma. By fitting a normal distribution function, a failure probability of 1.5 out of one million is estimated. However, the real distribution at the tail of the histogram is not known and estimating such a small failure probability with only 100 LHS samples may be strongly erroneous. Therefore, a reliability proof of the estimated safety level is required.

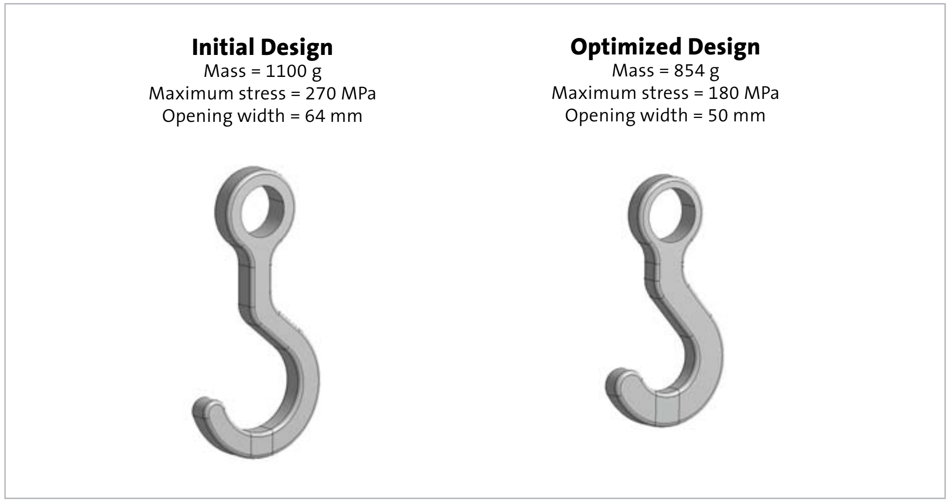


Figure 15: Initial and optimized design of the first deterministic optimization step

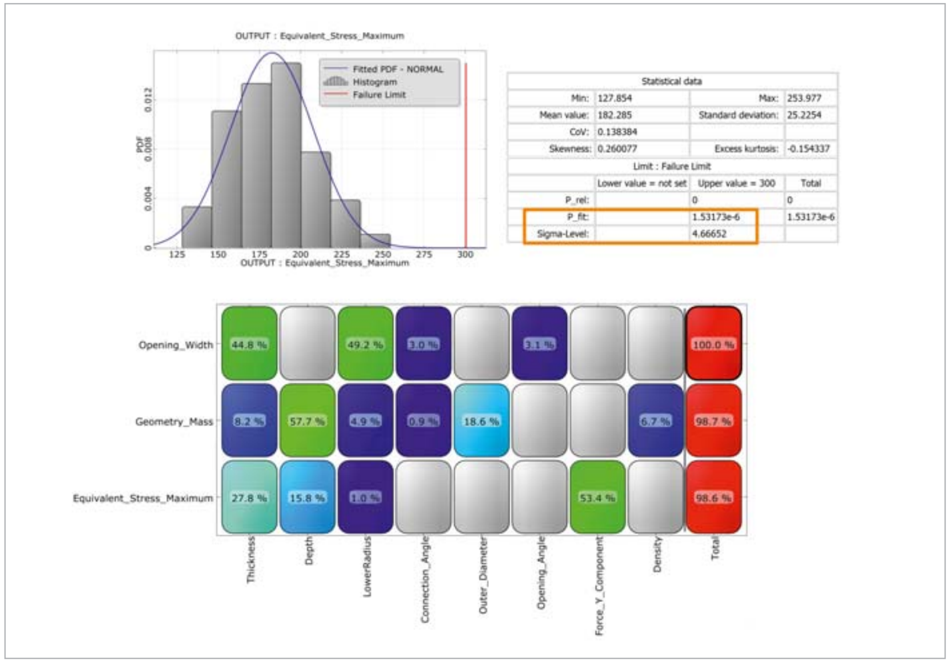


Figure 16: Results of the robustness evaluation of the first optimization step

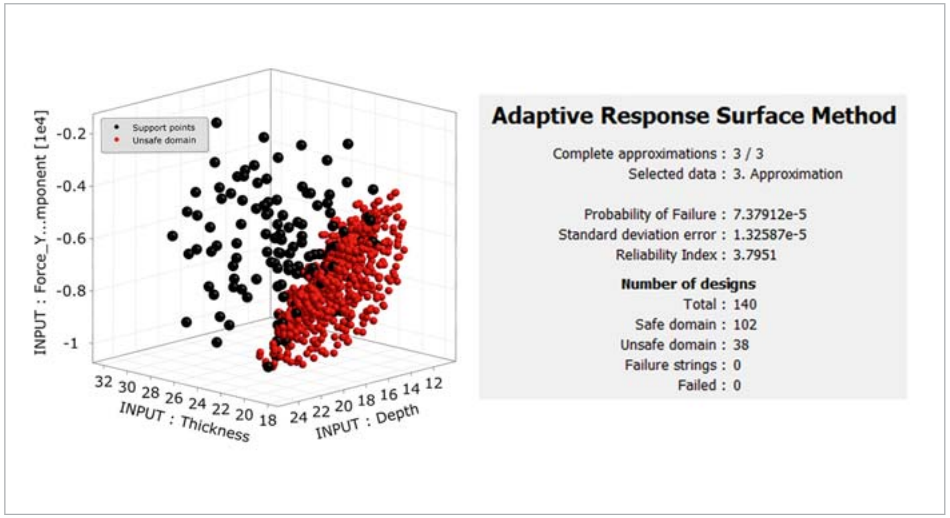


Figure 17: Support points and identified failure region of the Adaptive Response Surface reliability approach in the subspace of the most important input variables

Reliability proof of the design after the first RDO step

In this step, the failure probability shall be proven to fulfill a 4.5 sigma safety level. This is equivalent to a failure probability of 3.4 out of a million. The very robust Monte Carlo approach would need approximately 30 million samples to verify such a small probability with 10 % statistical error.

As reliability algorithm, the global Adaptive Response Surface Method with the Moving Least Squares approximation (Roos & Adam 2006) is used, which approximates the limit state function in the random variable space. In this analysis, only the important parameters as analyzed in the previous robustness sampling are considered. As indicated in Figure 16 (see previous page), only the thickness, the depth, the lower radius and the force component in the load axis are important with respect to the maximum stress. Starting with 100 LHS samples within a +/-6 sigma range, additional 40 samples are generated adaptively in the failure region. With these final 140 designs, a failure probability of 74 out of one million is estimated, which corresponds to a safety level of 3.8. In Figure 17, the support points and the detected failure region are shown. The figure indicates that only one dominant failure region exists. From these results, estimating a safety level of 3.8, it can be summarized that the estimation using the histogram from the robustness evaluation was too optimistic and the optimized design after the first RDO iteration still does not fulfill the robustness requirements. Therefore, a second RDO iteration step is necessary.

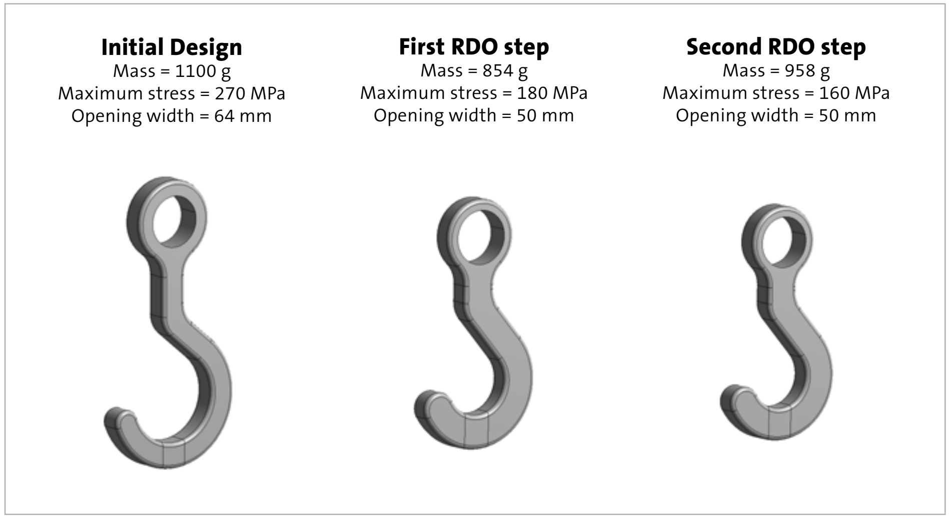


Figure 18: Optimized designs of the first and the second optimization step

Iterative Robustness Design Optimization – Second iteration step

Due to the violated robustness criteria of the first optimization step, the stress constraint condition is decreased to 160 MPa and the optimization is performed again. Using a start design of the initial sensitivity analysis which fulfills the modified constraint condition, the ARSM optimizer converges within 6 iterations using 89 designs. The obtained optimal design has the same geometric properties as from the previous optimization step but the depth is increased whereas the thickness is already at the upper bound. The optimal designs are compared to the initial one in Figure 18.

For the optimized design, a robustness evaluation is performed using 100 Latin Hypercube samples. The resulting scatter of the maximum stress is shown in Figure 19 (see next page).

Now, the estimated safety margin is 6.2 sigma based on these 100 samples and a fitted normal distribution, which is much larger than the required 4.5 sigma. However, the previous iteration step has shown that the roughly estimated safety level based on the variance estimates might have been too optimistic and, thus, a reliability proof is necessary. To prove the reliability, again a reliability analysis using the global Adaptive Response Surface Method by considering the four most important variables is performed. As indicated in Figure 20 (see next page), the estimated failure probability is less than one out of a million and corresponds to a reliability index of 4.8. There-

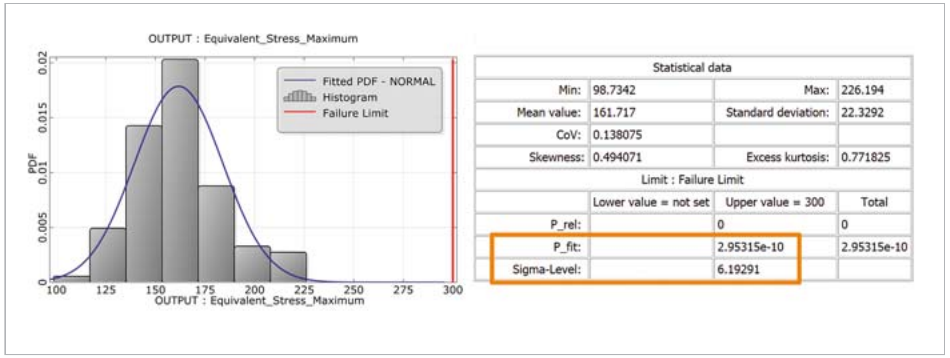


Figure 19: Results of the robustness evaluation of the optimal design from the second optimization step

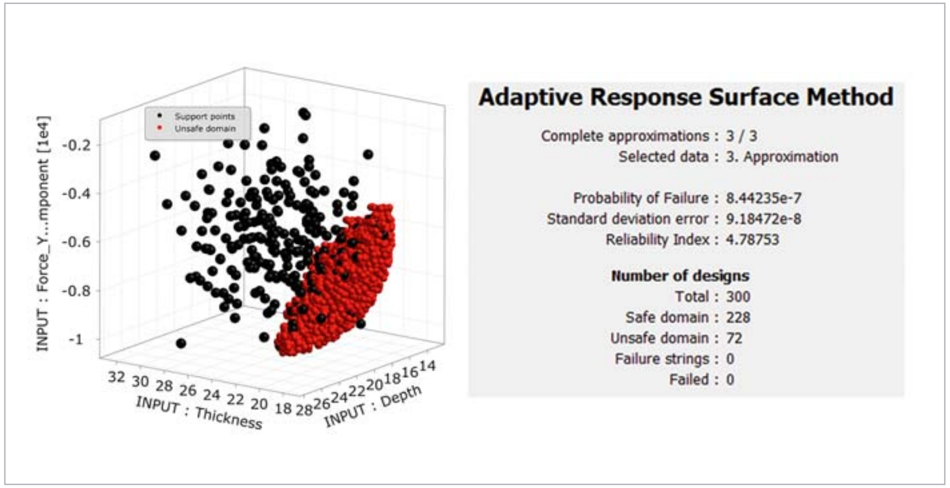


Figure 20: Support points and identified failure region of the Adaptive Response Surface reliability approach of the second optimization step

fore, it can be assumeed that the optimized structure fulfills the robustness requirements. However, as already mentioned in the chapter "reliability-based robustness analysis", it is strongly recommended to verify this reliability estimate with a second algorithm. For this purpose, an Importance Sampling using Design Point (ISPUD) method is applied, whereby the most probable failure point is obtained by a gradient based search within the FORM approach. In Figure 21, the results and the samples in the subspace of the most important parameters are shown.

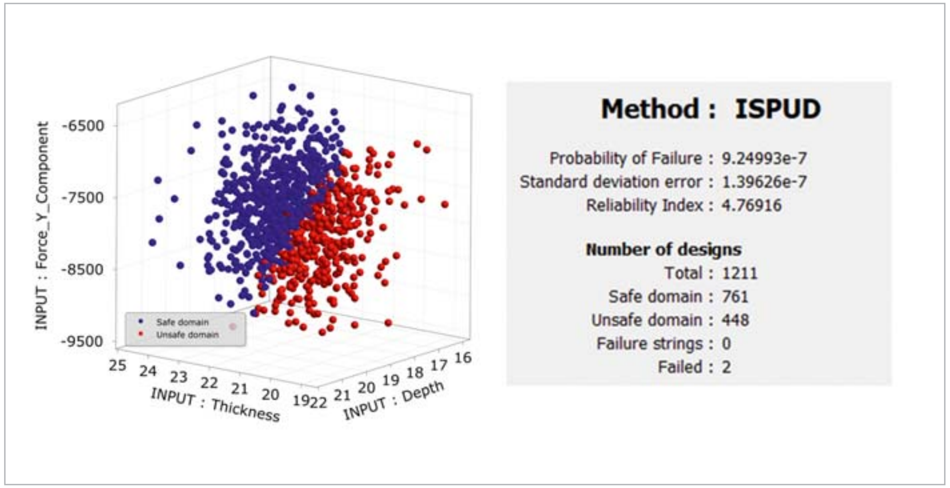


Figure 21: Final reliability proof of the the optimal design by the Importance Sampling approach

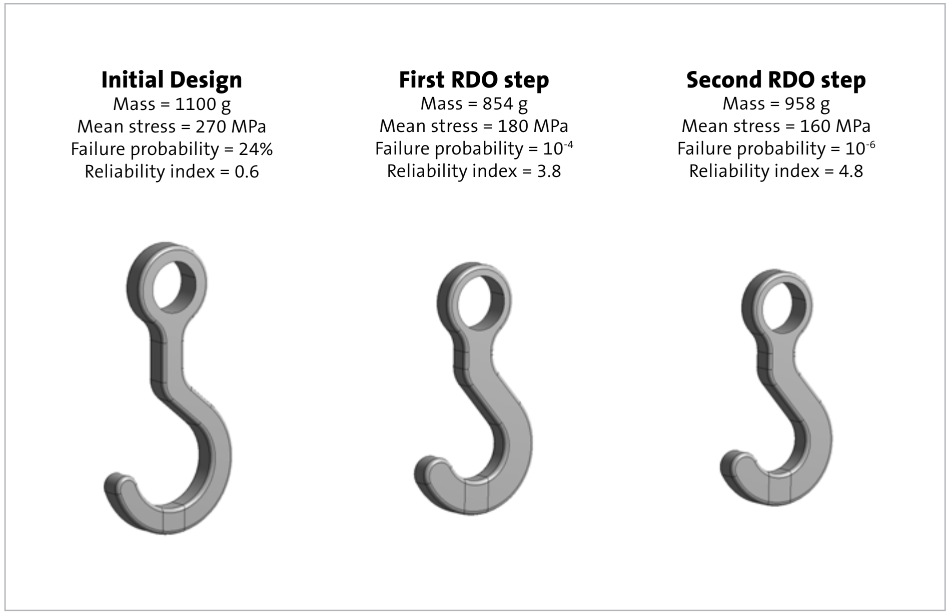


Figure 22: History of the iterative RDO of the hook example: starting from an initial design the safety factor is increased until the variance based robustness analysis and the reliability analysis indicate a robust design fulfilling the requirements

Summary of the iterative RDO approach

- Iterative optimization along a “Pareto front”: For each optimization step, the maximum stress in the deterministic optimization was reduced. As expected for the hook example, minimizing mass and minimizing stress in tendency are two conflicting objectives. Also, it is interesting to see that the variation of the maximum stress as a result of constant production and loading uncertainties also rises with lighter structures. There is the phenomenon that optimized structures often tend to be more sensitive to uncertainties which motivates the integration of robustness evaluation in virtual prototyping.

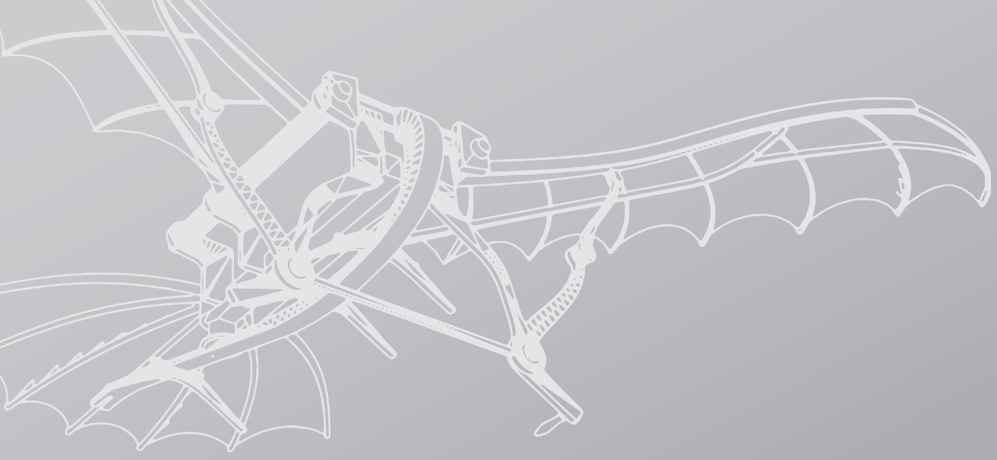
- The optimal design of the first optimization step using a global safety factor of 1.7 to the stress limit exceeds the failure limit with a 4.7 sigma safety margin. The variance based estimation of the second design indicated a 4.5 sigma design, but the reliability analysis did show that the estimation was too optimistic. After the seond optimization step with a further increasing of the safety factor to 1.9, the design was proven to be a 4.5 sigma design having a failure probability lower than 3.4 out of a million design realizations.

SUMMARY AND CONCLUSIONS

An initial sensitivity analysis is very useful in both the optimization design space as well as the scattering variable design space. Whilst any dimensionality of design space is valid for direct optimization, for real world RDO applications it has to be expected that (at least in the robustness space) to start with a large number of potentially important scattering variables.

In contrast to the design space of optimization, the variable reduction in robustness space starting from all possible influencing variables is only possible with deep knowledge of the relative importance of the scattering variables. The initial sensitivity analyses are therefore crucial for the selection of the appropriate optimization and stochastic algorithms. Furthermore, they are essential for an appropriate reducing of the task size to the most effective optimization variables and the most important scattering variables.

If the RDO task is defined with appropriate robustness measures and safety distances, multiple optimization strategies can be performed successfully. If a design evaluation needs significant time, the balance between the number of CAE design runs and the accuracy of robustness measures is a challenge for all RDO strategies, iterative or simultaneous. All approaches attempt to minimize the number of design evaluations and to estimate the robustness measures. If small failure probabilities (less than 1 out of 100) need to be proven, algorithms of reliability analysis have to be applied, at least at the end of an RDO process to prove the optimal design.



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