Simulation of Fracture Design Generation, Production Characteristics and Temperature Development of a Hot Dry Rock Geothermal Reservoir

Animesh Ranjan, Stefan Eckardt and Johannes Will

Dynardo GmbH

Abstract

The paper focuses on simulation of the fracturing process during generation of the heat exchanger and the post-fracturing during production of a Hot Dry Rock (HDR/EGS) Geothermal reservoir. A general EGS reservoir has typically three cycles, namely, Hydraulic Stimulation, setup of the power plant with Reservoir Relaxation and Reservoir Production. The usability and economic efficiency of a HDR reservoir depends on the generated heat exchanger area, the location of stimulation and production wells and the temperature development over a specified number of years. The basis of the simulation approach is a homogenized continuum modelling of the physical phenomena prevalent in the various cycles of the reservoir operation. Based on Dynardo's 3D hydraulic fracturing simulator, the stimulation process is integrated with the relaxation and the production cycle to estimate characteristics such as heat exchanger area, connected height of fractures and pressure losses. Additionally, a thermal cycle has been developed to estimate the temperature distribution over a period of up to 60 years. Considering the uncertainties in procuring reservoir data for the simulator, the study has been backed by a Sensitivity study to operational conditions and to uncertain reservoir conditions.

Keywords: Geothermal Simulation, Enhanced Geothermal Systems, Thermo-hydro-mechanical Analysis, Hot Dry Rock Reservoir

1 Introduction

Energy consumption in the world has seen an ever increasing upward trend since the early part of the 20th century. Still a major chunk of the total energy supply comes from non-renewable energy resources with approximately 66% supplied only by crude oil, natural gas and coal reserves. The large share of these resources are not just restricted to developing economies such as China, Brazil, etc but also economic powerhouses such as USA and the European Union. Geothermal Energy provides tremendous benefits in terms of reducing dependence on fossil fuels, reducing greenhouse emissions and generating new economic and employment opportunities. Geothermal Power systems aim to extract the inexhaustible heat available beneath the earth's surface. Natural Geothermal springs are rare to find and hard to locate and as could be used supposedly as an alternative to reducing dependence on naturally occurring hydrothermal reservoirs. EGS reservoirs are set-up by drilling wells beneath the earth's surface and creating an artificial permeable fracture network between the wells. In that process Hydraulic Fracturing is a well-stimulation technique used for creating artificial heat exchanger by creating a network of permeable fractures between injection and production wells.

During the last 15 years, Dynardo GmbH has developed a) Thermo-Hydro-Mechanical (THM) simulation environment based on ANSYS® implicit finite element code for parametric FEM modeling for geotechnical applications with successful applications in Dam Engineering [1], Hydraulic Fracturing of unconventional Oil, Gas reservoirs [2], Geothermal reservoirs [3] and Nuclear Waste disposals, b) Dynardo's toolbox for parametric variation optiSLang® for sensitivity analysis and calibration of large number of reservoir parameters as well as engineering and operational conditions parameters and their influence on the final stimulated rock volume, the production and their uplift potentials. Since 2008, the THM simulation capabilities have been extended to 3D simulation of Enhanced Geothermal Systems or Hot Dry Rock Geothermal reservoirs.

The Fracturing design in an EGS reservoir is different to fracturing design in an oil or gas reservoir. The process, more commonly referred to as Hydro-shearing [4] aims to generate mainly shear fractures between rocks by inducing shear failure in contrast to Hydraulic Fracturing in oil or gas reservoirs where tensile fractures are targeted, with injected fluid used to break the rock along with a proppant mixture are used to maintain the created openings. For EGS applications, integration of the Production Cycle, which directly follows the fracturing cycle and a prescribed unloading time in a stimulated reservoir, is an open task. During the Production cycle, there is a significant pressure loss between injection and production well due to flow resistance in the reservoir. This flow resistance also referred to as the impedance is a function of the fracture opening which further depends on overall pressure levels in the fracture network, temperature of surrounding rock and other geo-physical effects such as sliding of fracture planes along the faces of the fracture surface [5]. In this paper, the various aspects of EGS simulation such as Stimulation, Relaxation and Production cycle and their associated thermal-mechanical-fluid aspects and interactions are discussed.

2 Hydraulic Stimulation Cycle

The hydraulic fracturing simulator for the 3-dimensional simulation of the hydraulic fracturing process is based on coupled hydraulic-mechanical finite element analysis as shown in *Fig. 1*.

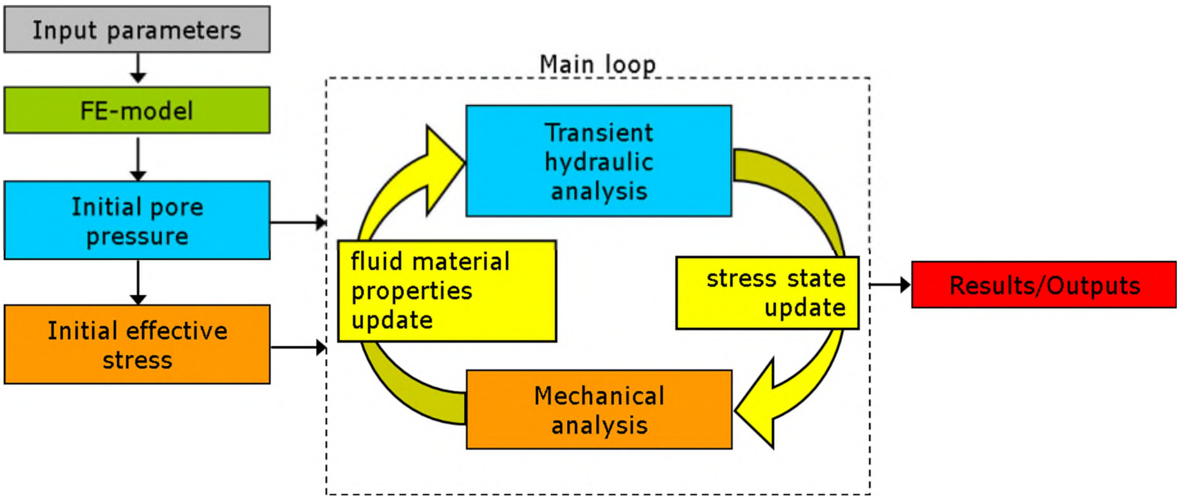


Fig. 1 Coupled hydraulic-mechanical fracturing simulator

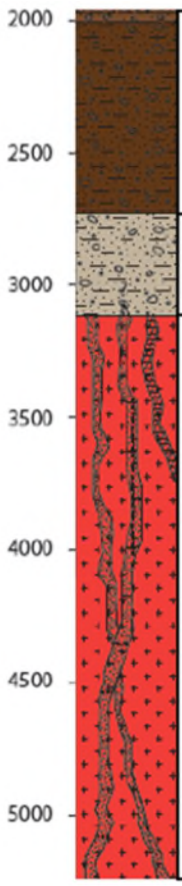
The main features of the fracturing simulator are as listed below [6]:

i. Non-linear mechanical analysis using multi-surface plasticity for modelling fracture network activation in jointed rocks within homogenized continuum approach [7].
ii. Hydraulic model is based on the assumption of laminar flow in multiple parallel joint systems [8].
iii. The mechanical to hydraulic coupling which involves computation of fracture opening and closure resulting in anisotropic hydraulic jointed rock conductivity.
iv. The hydraulic to mechanical coupling which involves computation of flow forces, depending on the pressure gradients within the jointed rock.

v. Very important to realistically simulate the non-linear history of fracture network creation and activation is the initialization of reservoir conditions, the initial in situ strength, stress and pore pressure conditions.

2.1 Meiningen Suhl Reservoir Model

The simulation of the heat exchanger generation in the Meiningen - Suhl reservoir model is based on a 1 Well - 3 Stage model. Post calculation of 3 stages in the reservoir model, the effect of further stages is extrapolated. The landing depth of the stimulation well is defined at 4500 m. A schematic is illustrated in figure 2. The conglomerate formation beyond 3000 m is not modelled in the FE-environment in order to reduce FE mesh size, assuming that vertical fracture extension is no longer than 1500 m.



Ilmenau Formation	Sand and Sandstone conglomerates, Black shale with stromatolitic carbonates and other Tuff boulders and Lapilli formations.
Möhrenbach Formation	Sand and Sandstone conglomerates, Black shale with coal layers
Primary Granite – Thüringen Formation	Granite formation : Colours – light – grey, red and gray-red formations. Major Joint orientation : Northwest – Southeast, Northeast – Southwest, North – South. Quartz and Calcite formations at scattered locations. Reservoir Temperature : 130°C between 3700 – 3900 m, 140° C between 4000 – 4250 m

Fig. 2 A schematic of reservoir layers in Meiningen/Suhl (DBI Input Sheet, 2013 **[9]***).*

The stimulation process is carried out in several stages. Based on the reservoir permeability and in-situ stress and strength conditions, the stage design needs to be optimized. The parametric model is designed to use reference points for each stage, see figure 3. The beginning, middle and end points refer to individual stage locations in space, in relation to which the stage definition is established. Definition of a single point, i.e. 'beginning', 'middle' or 'end' along with the well orientation or drilling direction and the perforation length can be used to model a given well-stage location. It should be noted here that the current study assumes a single-perforation model, i.e. each stage has only one perforation through which the fluid is pumped in.

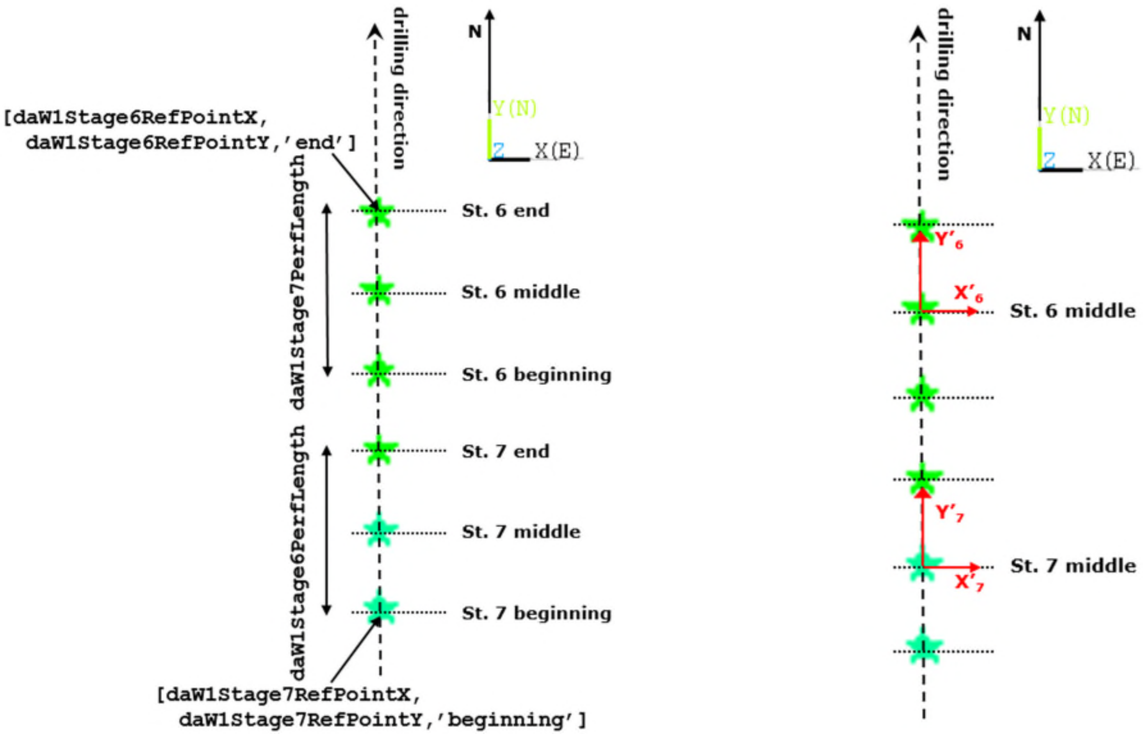


Fig. 3 Definition of Stages using Reference Points along the Drilling direction.

2.1.1 FE Model and Parameters

A 3D Finite Element model is developed and investigated in this study [10]. The models are illustrated in figure 4.

Associated with the model, are reservoir and operational parameters employed during the simulation. Reservoir parameters denote specifications which are either characteristic to the reservoir layers, such as the initial stress state of the reservoir or the initial permeability. They also include the strength definitions of the intact or jointed rocks. Operational parameters, on the other hand, specify reservoir loading conditions or the operational characteristics of the well, fluid employed, etc. Table 1 illustrates some of the parameters used in the Meiningen – Suhl model to investigate the influence of reservoir uncertainties and operational parameters to optimize the well and the hydraulic fracturing design.

Parameter	**Value**	**Units**
Operational Parameters		
Water Injection Rate	0.0833	m^3/s
Injection Time per Stage	1000	min
Dynamic Viscosity	1	cP
Well Azimuth	70.0	°(deg)
Reservoir Parameters		
Initial Pore Pressure Gradient	10.66	kPa/m
Total Vertical Stress Gradient	35.10	kPa/m
Minimum Total Horizontal Stress Gradient	29.80	kPa/m
Maximum Total Horizontal Stress Gradient	60.90	kPa/m

Table 1 Reservoir & Operational Parameters for Meiningen/Suhl Reservoir Model

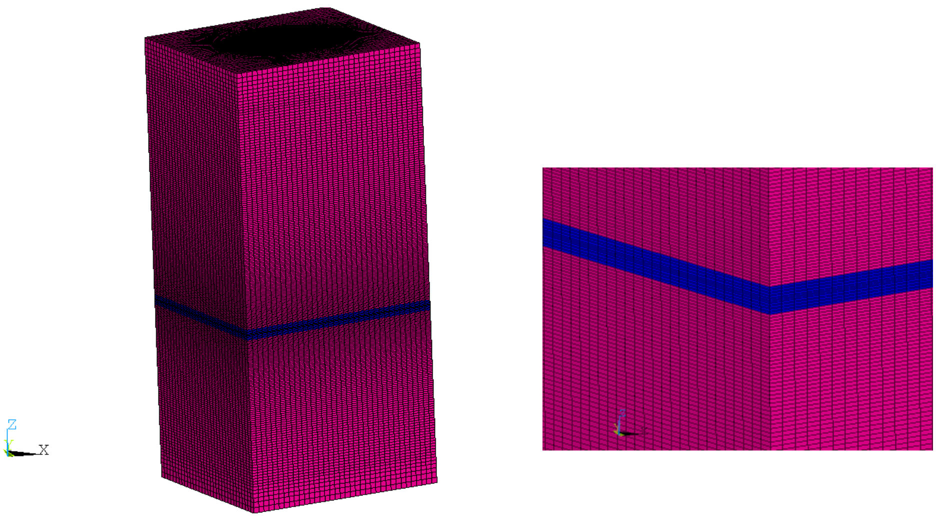


Fig. 4 View of the hydraulic model along with the zoomed view showing the fine mesh regions around the perforation element.

3 Generation of Heat Exchanger with Hydraulic Stimulation

The fracturing or stimulation cycle is used to generate the heat exchanger, upon which the Production cycle is further carried out. Moreover, simulation of the fracturing cycle gives a good initial impression about the accessible potential of the reservoir. Compared to most other reservoir simulators, where the characteristic dimension of the heat exchanger needs to be provided before-hand, Dynardo's simulator with its capability to deal with the anisotropic in-situ stress conditions and anisotropic strength conditions of jointed rock works directly on intrinsic reservoir parameters. After pumping the 1 Well – 3 Stage model with a stimulation rate of 0.0833 m^3/s, a fracture network is generated which represents the design of the heat exchanger.

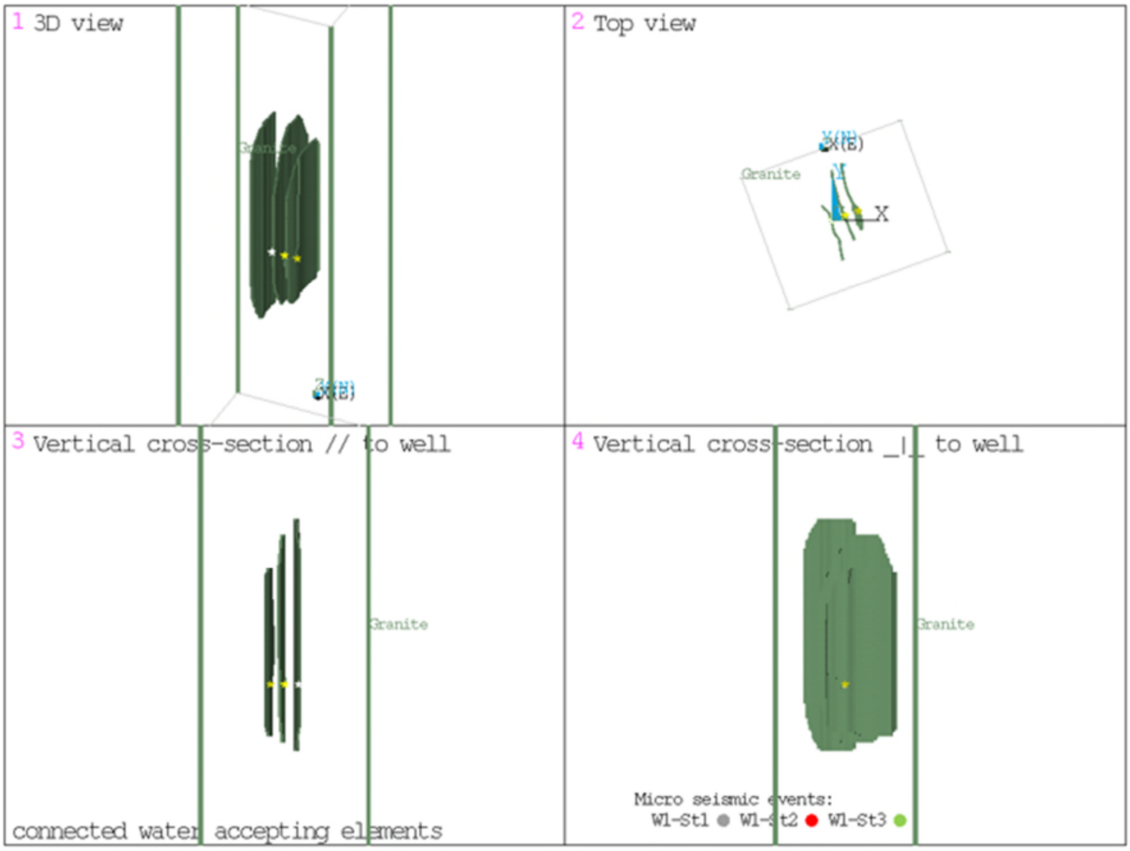


Fig. 5 View of the Fracture Core representing the Heat exchanger generated during Hydraulic Stimulation

Fig. 5 shows the resulting fracture planes after stimulation of the three stages. The total amount of fluid injected into the reservoir and the resulting bottom hole pressure levels in the fractures is illustrated in figure 6.

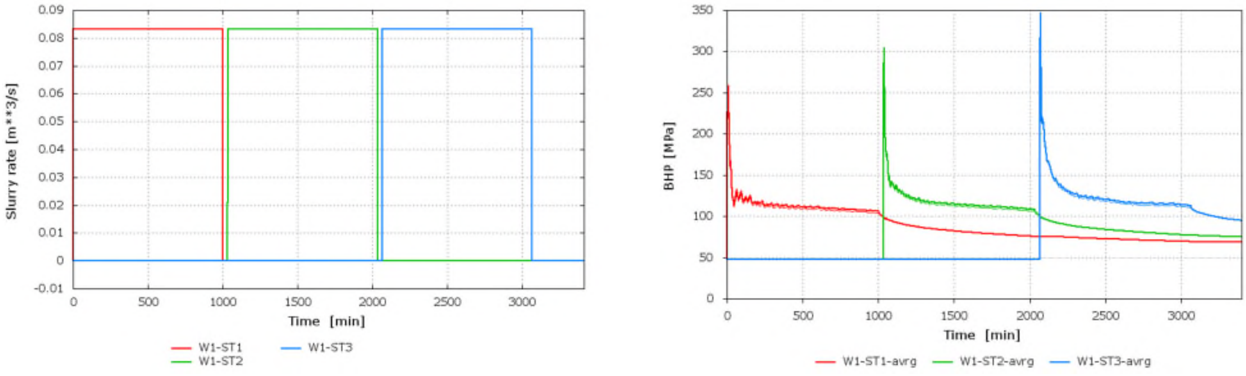


Fig. 6 Injected Slurry Rate into the reservoir and resulting bottom hole pressure responses

As the specified slurry is injected into the reservoir, it leads to a sudden pressure increase and after fracture initialization over the course of further pumping and growing fracture height the pressure values slightly decrease. Note that the peak pressure is influenced by mesh size and does not represent any physically meaningful value. As the subsequent stages are pumped, the previous stages tend to further relax. The volume distribution and volume balance between created fracture volume and injected fluid volume in the reservoir are an integral part of the numerical calibration of the uncertain reservoir parameters. Apart from calibration of fracture initiation & termination pressure conditions, bottom hole pressure response with measured BHP and stimulated rock volume with micro-seismic data, the generated fracture volume is calibrated with the pumped total fluid volume. The total fracture volume is compared with the total pumped fluid volume, post the aforementioned rate and pressure calibration. The total fracture volume is calculated based on the mechanical openings and considering low matrix permeability values of granite rocks, it should be close to the pumped fluid volume. Storage in joints and leak – off volumes represent the total amount of fluid stored in fractured elements in the hydraulic domain and the fluid volume that is eventually lost and cannot be used for fracture growth. The volume balance is highlighted in fig 7.

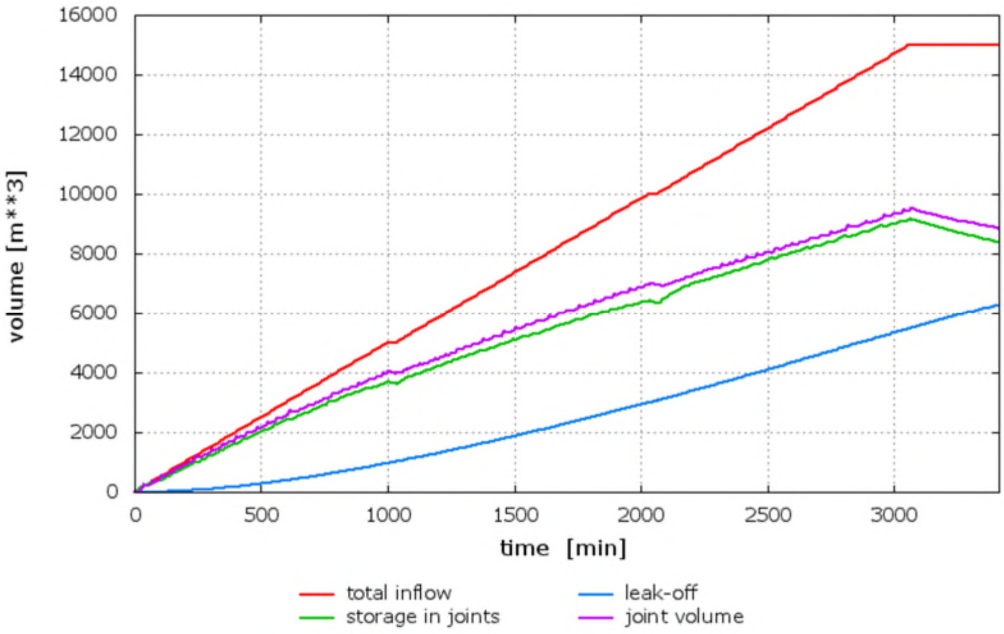


Fig. 7 Volume Balance in Reservoir – Meiningen/Suhl

4 Reservoir Relaxation and Production Cycle

The unloading interval which follows the fracturing process, allows the pressure in the fracture system to relax. The unloading process is directly followed by the production cycle, where an applied slurry is injected into the production well to allow it to permeate through the fracture network and get heated up in the process. In order to optimize the heat exchanger the landing of the production well needs to optimized. An optimal production well orientation depends on the total extension of the fractures, orientation of the fracture network and the conductivity of the fractures. The orientation of the injection well and the stress gradient in the reservoir also play a big role in determination of the optimal position and orientation of the production well.

In practice, the placement of the production well is ambiguous wherein the timing of the production well drilling is not clearly demarcated. Introducing the Production well after the stimulation procedure should be a more profitable choice since the probability of connecting the two wells with a high permeable fracture network would be much higher. The available heat exchanger area would be hence maximized after verification of the established fracture system, generated during the stimulation process. However, this depends primarily on the uncertainty variation of characteristic reservoir parameters. In order to estimate the most optimum production well position, best available measurements about fracture extension and orientation (such as micro-seismic measurements (MSE) during stimulation process) and best available simulation tools used to forecast and calibrate the fracture extension and conductivity need to be combined. In the current study, the well selection algorithm is based on the practical assumption that the dip and orientation of the production well is similar to the injection well in use and the heat-exchanger area is maximized.

4.1 Unloading Cycle – Reservoir Relaxation

The simulation of the unloading cycle is aimed at replicating the time-gap between the fracturing and production cycles during setup of the power plant. Unloading is scaled in time by applying a pressure boundary condition to the injection well corresponding to the initial pore-pressure at the given depth. The loading is ramped in order to avoid sudden gradients and numerical errors in the model. In order to ensure zero inflow during unloading, the applied pressure gradient must be larger or equal to the recorded bottom hole pressure gradient at the end of stimulation step. The rate of relaxation of the stages depend on the conductivity and pressure level of the individual fractures. This is clearly illustrated in figure 8 where Stage 3 fracture shows larger out-flow rates compared to the other two fracture networks since stage 3 has a higher pressure level at the end of the stimulation cycle.

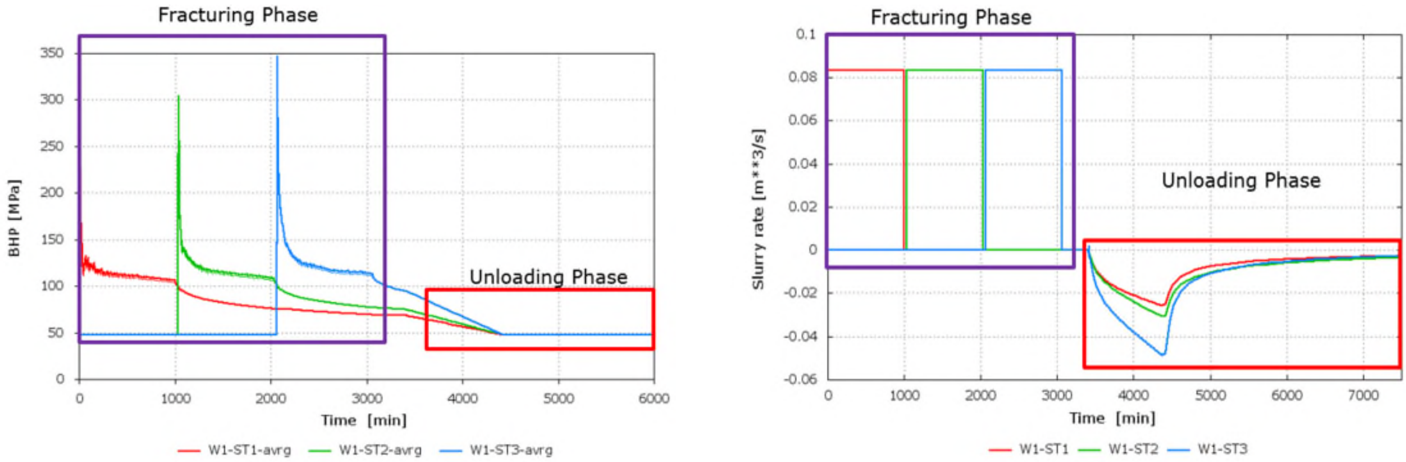


Fig. 8 Fracturing + Unloading - Pressure Gradients & Slurry Volumes

4.2 Production Cycle

The simulation of Production cycle in an EGS reservoir has several aspects, characterized by implementation of the cycle in Dynardo's hydraulic fracturing simulator. These aspects include numerical algorithm implementation for selection of Production wells, modelling reservoir flow resistance and determination of volumetric efficiency and/or leak – off in reservoir.

4.2.1 Production Well Selection Algorithm

The objective of Production well algorithm is estimation of maximum heat exchanger area for the reservoir. The optimal location is based on height and orientation of generated fractures. A numerical algorithm has been developed considering dip and orientation of injection well, hydraulic conductivity of generated fractures and pressure change in the reservoir. Based on the algorithm and number of fractures in the model, three possible well scenarios have been illustrated in figure 9.

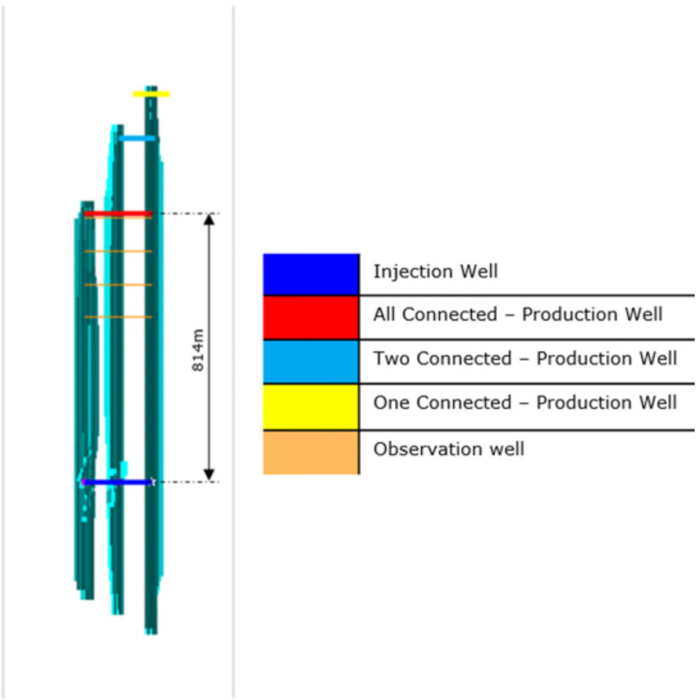


Fig. 9 Stage Connection: Well Connection scenarios

Based on the maximum heat exchanger area calculated with the possible well connection scenarios (figure 6), one of the above well connection scheme is automatically selected. In the current Meiningen – Suhl model, it corresponds to a height of 814 m.

4.2.2 Reservoir Flow Resistance & Leak – off

Pressure loss and fluid volume loss constitute two of the primary losses encountered in EGS Production cycle, which could have an adverse impact on reservoir economics. The expected change in pressure level in the Meiningen/Suhl reservoir 20-30 bar, which is considered as the target value for the reference numerical simulation. In order to reproduce pressure changes in the reservoir during production cycle, an equivalent system viscosity could be considered which doesn’t represent the pumped fluid velocity but an equivalent reservoir resistance value. For the Meiningen/Suhl reservoir, pressure value of 22.4 bar is calibrated at 1 cP system viscosity, which in this case is also equal to the pumped fluid viscosity (see figure 10).

Leak – off estimation refers the fluid volume lost in the reservoir and did not contribute to energy generation in the Production phase. Permanent loss of circulation needs to be replaced and might become a cost factor to the production. With relevance to the Production cycle, leak off volume represents the amount of fluid dissipated in the surrounding formation rock from the onset of the Production cycle. After initial stabilization of dynamic effects, a leak – off gradient is estimated since leak – off volume has a fairly linear relationship with Production Time (see Figure 11)

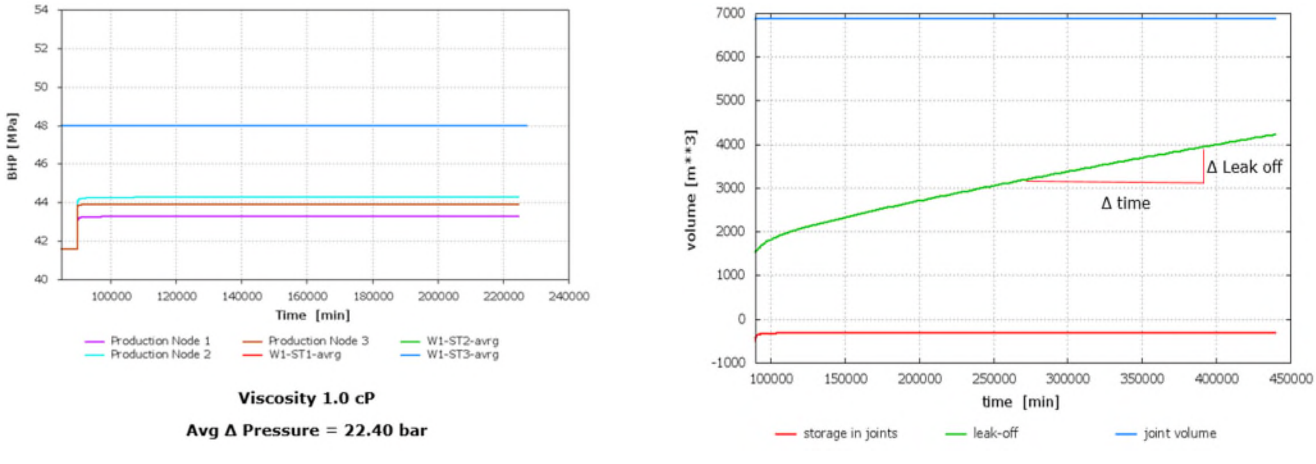


Fig. 10 Pressure Loss and Leak off Estimation

4.3 Sensitivity Study

The sensitivity study is aimed at estimation of the variation of all relevant system responses as a result of variation of operational parameter and/or reservoir uncertainties. The design space is defined using windows of reservoir uncertainties and operational parameters, see Table 2. In order to identify the correlation structure between input variation and response variation, meta-modelling is performed and the predictive ability of the best possible generated meta-model for response variations is estimated using the software optiSLang [11] based on a forecast measure, known as Coefficient of Prognosis (CoP) [12]. The meta-model of optimal Prognosis quality (MOP) and the related CoP are highlighted in figure 11.

Parameter Name	Reference Value	Minimum Value	Maximum Value
Stage Distance [m]	100	100	200
Well Azimuth [°]	70	40	100
Well Dip [°]	0	-20	0
Pore Pressure Gradient [Pa/m]	10660	5000	11000
Youngs' Modulus [Pa]	7.8E+10	6.5E+10	8.5E+10
Friction Angle [°]	41.8	35	42
Uniaxial Compressive Strength [Pa]	1.43E+08	1.1E+08	1.8E+08
Dilatancy Angle [°]	20	15	25
Relative Joint Stength	0.16	0.1	0.3
Vertical Stress Ratio	0.8	0.75	0.85
Horizontal Stress Ratio	6.41	6.0	7.0
Slurry Volume [m^3/s]	5000	2500	7500
Stimulation Rate [m^3/s]	5	5	10
Production Rate [m^3/s]	100	80	120
Horizontal Well Length	1500	1000	2000

Table 2: Design Space Parameters and Ranges

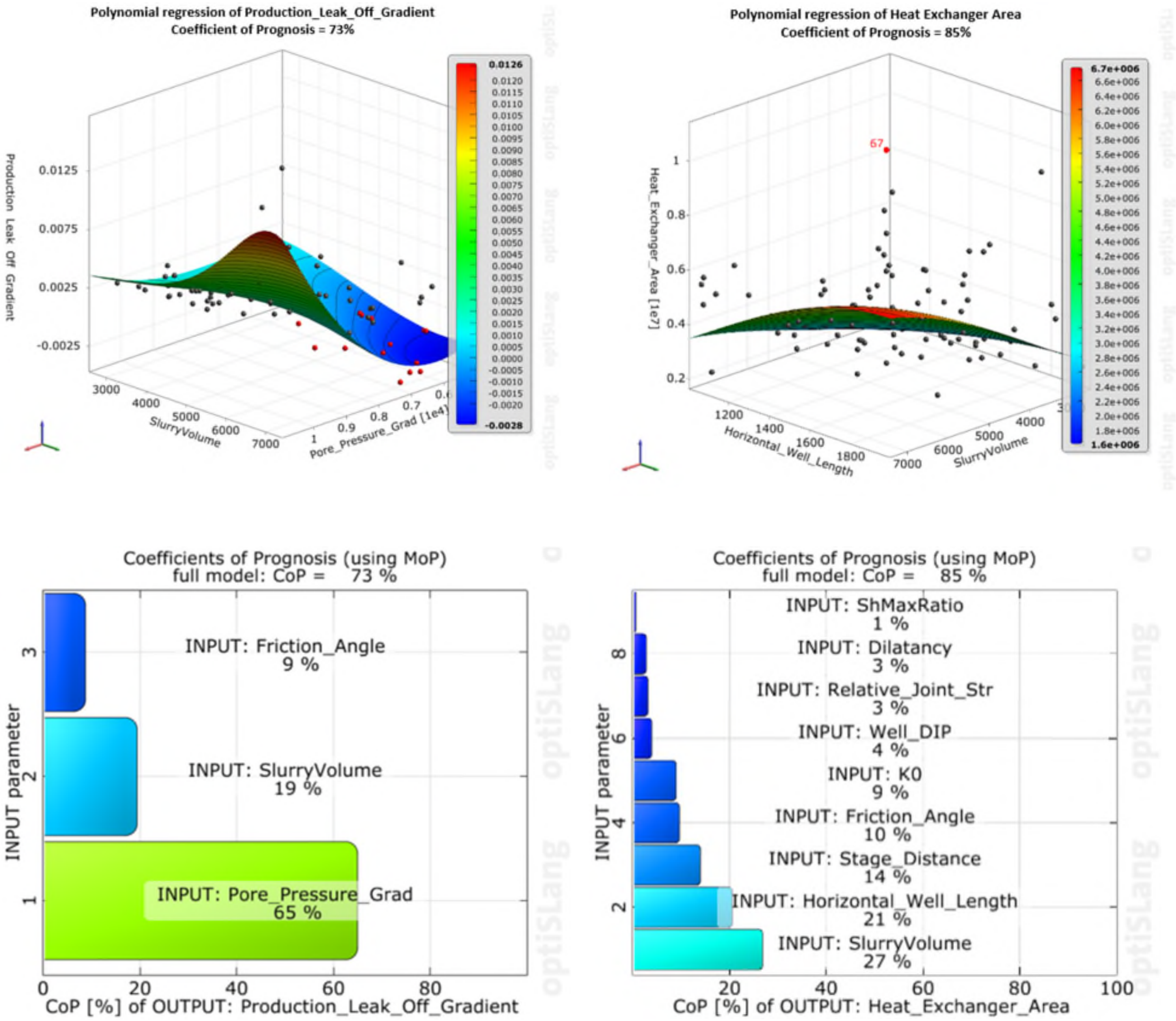


Fig. 11 Metamodel and COP for Leak-Off and Heat Exchanger Area

Based on the largest heat – exchanger area, design 67 was chosen as the best design and it has been compared with the reference design in table 2.

Response Name	Reference	Design 67
Connected Heat Exchanger Area [m^3]	0.62E7	1.15E7
Connected Height [m]	831.0	1198.12
Average Pressure Change – Production [bar]	22.45	35.52

Table 3 Comparison of Results: Reference Design versus Best Design

5 Coupled Fluid-Thermal Simulation

The physical principle of Production Cycle in an EGS reservoir is dominated by a coupled fluid – thermal simulation, also referred as Conjugated Heat Transfer (CHT) Analysis. The main characteristic of a CHT simulation is the accompanying heat transport where the dominant mode of heat transport is convection. The transport equation is represented by (1) as follows:

$$\rho c_p \left(\frac{\partial T}{\partial t} + \vec{u}.\nabla T \right) = \nabla.(k\nabla T) + Q_h \quad (1)$$

where ρc_p - Overall heat capacity of medium [J/m^{3}K],

T - Temperature [K],

t - Time [sec],

$\vec{u}$ - Fluid velocity vector [m/s],

k - Thermal conductivity [W/m K] &

Q_h - Heat source/sink [W/m^3]

Due to application of the homogenization principle, the application of fluid velocity is restricted to modelling of a Darcy velocity which is represented by the resulting fluid flux of the groundwater equation. Since the transport phenomenon is dominated by a steady fluid flow in the reservoir, available commercial CHT tools are not ideally suited for the current application using a homogenization modelling approach since they cannot handle fluid and thermal interactions independently. Instead, it is solved using elements with numerical formulations allowing for coupled fluid-heat transfer with time – independent velocity vectors and an additional in-house “Artificial Diffusivity” [13] algorithm providing stability to the solver in convective – flow scenarios.

5.1 Continuum Homogenization and Lauwerier Problem

Continuum homogenization of material and phenomenological parameters has been implemented in the hydro – mechanical fracturing cycle of the EGS process. The same principle is extended to the coupled hydro-thermal Production cycle with homogenization of thermal parameters such as thermal conductivity and overall heat capacity being based on the defined porosity of the permeable rock. The homogenized parameters are obtained from the following relationships:

$$\rho c = \left(\varphi \rho_{fluid} c_{fluid} + (1 - \varphi)\rho_{rock} c_{rock}\right) \quad (2)$$

$$\lambda = \left(\varphi k_{fluid} + (1 - \varphi) k_{rock}\right) \quad (3)$$

Where,

λ - Heat conductivity [J/(msK)]

ρ - Density [kg/m^3]

c - Specific heat capacity [J/kgK]

φ - Porosity

In order to arrive at a process chain for simulating the coupled fluid – heat phenomenon in an EGS reservoir, the established methodology is tested with a groundwater model with available analytical solution [14]. The analytical model and the corresponding numerical model is illustrated in figure 12.

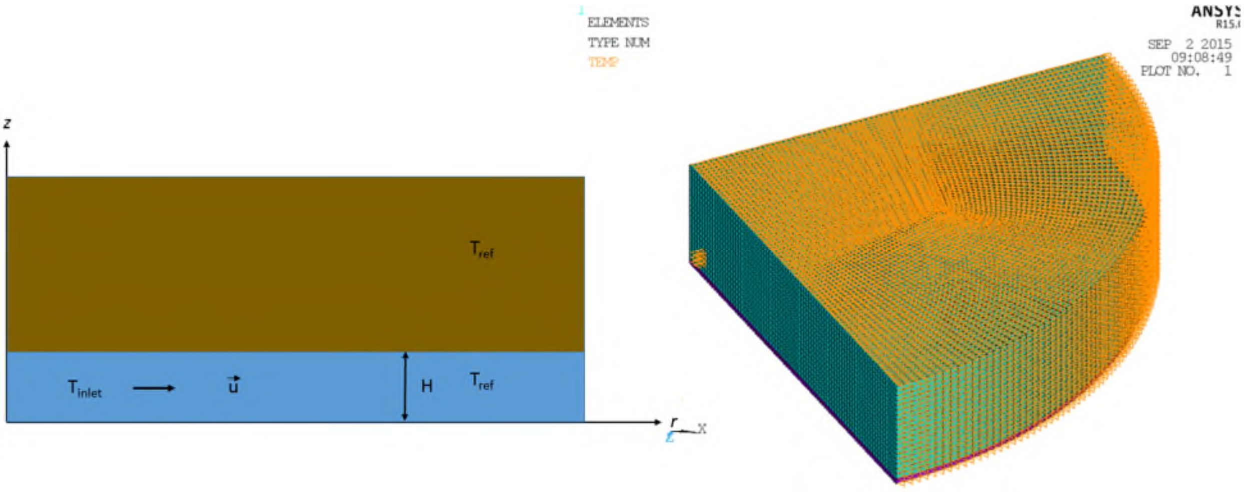


Fig. 12 Analytical and Numerical Model for Heat transfer in Porous Media

The numerical model was tested for a range of fluid flow rates between 10 L/s and 100 L/s with 2D and 3D axisymmetric and quadratic symmetry models. The fluid is injected at a temperature of 80^{o}C with rock having an initial temperature of 150^{o}C. In order to validate the energy modes in the numerical model, an energy balance has been developed. The energy balance takes the various physical energy exchanges within the model into consideration. It is, however, essential that the total energy in the system is conserved throughout the heat-exchange process. The results are highlighted in figure 13.

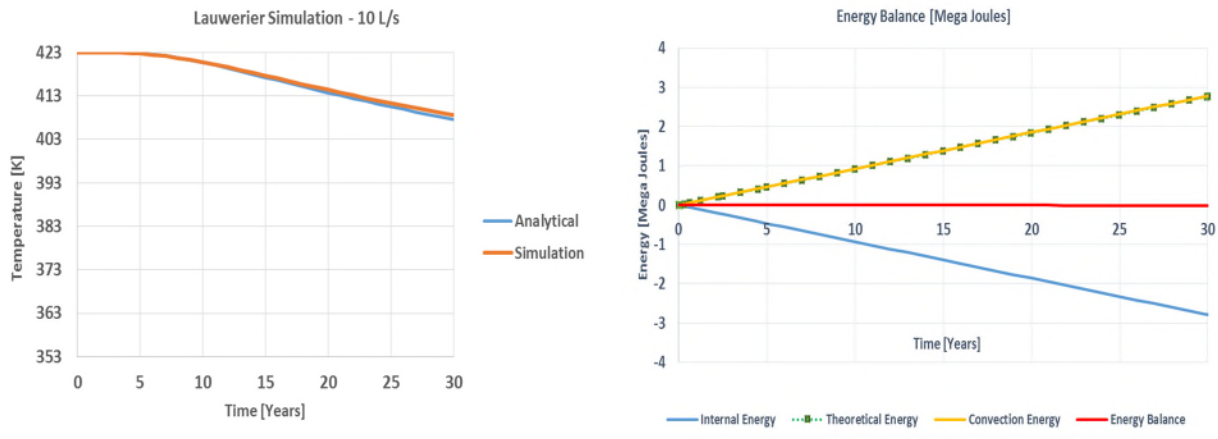


Fig. 13 Outlet Temperature (Simulation v/s Analytical) and Energy Balance (10 L/s)

With the numerical model having been validated with the Lauwerier Problem, the approach is tested for the 3D unstructured Meiningen – Suhl model. The Darcy velocity obtained after Fracturing and corresponding Relaxation time is extracted and simulated with the developed coupled fluid-heat algorithm with Artificial Diffusivity. Initial conditions of the model include introduction of a temperature gradient equal to -0.0214 ^{o}C/m, with injected fluid temperature at 80 ^{o}C. The temperature profile at the end of 60 years and resulting power extraction over a 60 – year period for injected fluid rate of 10 L/s is illustrated in figure 15.

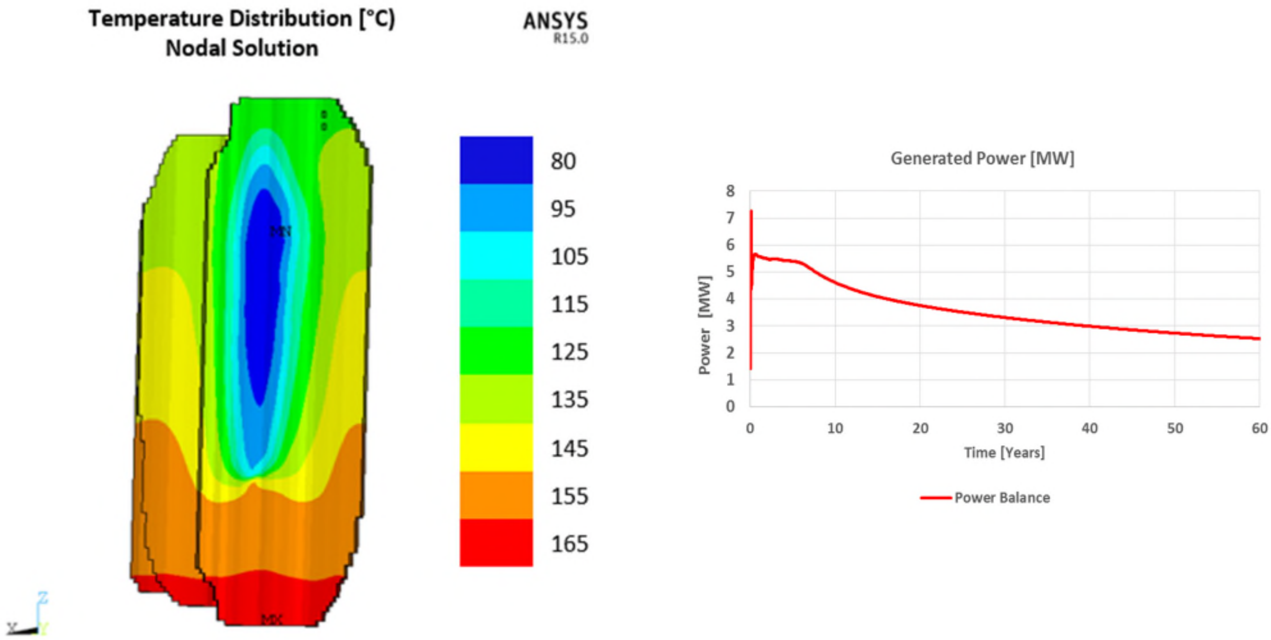


Fig. 14 Temperature Profile and Power Distribution for Meiningen – Suhl Model (10 L/s)

6 Conclusions and Future Work

The current study focuses on development of an integrated numerical simulation process chain for a Geo-thermal heat exchanger generation and operation cycle. The objective of the simulation process chain is to provide forecast quality predictions for the complete EGS process chain including the Design phase - for optimization of heat exchanger area, Reservoir Drilling and Stimulation phase - in order to calibrate the reservoir model and Decision phase - concerning location of Production well and simulation of Production phase. A reference model based on reservoir conditions in Meiningen/Suhl, Thüringia, Germany has been evaluated. The simulation technique is a combination of several standalone individual processes, namely, stimulation phase, unloading phase and production phase. The basic building block of the numerical simulator are the 3D fracture simulation, 3D Darcy-Flow equations in jointed rock and the general 3D advection-conduction equation. Development of an additional thermal simulator, based on heat transport formulation in ANSYS, has been carried out which has further extended the process chain beyond its original scope. The thermal simulator has been validated with available analytical solutions. In order to optimize the design of the EGS system, a Sensitivity study has been carried out; based on variation windows of uncertain reservoir and operational parameters. Consequently, the best design parameter set along with the sensitive parameters have been identified. As a last step cost function will be introduced in the overall optimization cycle. Than the design and optimization under generation and production of EGS systems will be performed in an integrated software environment.

7 Acknowledgments

The authors would like to thank the Thüringer Aufbaubank for the financial support within the framework of the joint research project OPTIRISS. The authors would also like to express their heartful gratitude to Optiriss' project partners, namely, the engineering team at Jena-GEOS and the Department of Earth Sciences, TU Bergakadmie Freiberg for supporting the research work by providing valuable data and information.

8 References

[1] B. LANGE, F. HERING, R. SCHLEGEL, G. LEYENDECKER and F. ORTEGA, "HRB WALDBÄRENBURG: First RCC Dam Experience in Germany," in *6th International Symposium on Roller Compacted Concrete (RCC) Dams*, Zaragoza, October 2012.

[2] J. Will, "Optimizing of hydraulic fracturing procedure using numerical simulation," in *Weimar Optimization and Stochastic Days*, Weimar, 2010.

[3] J. Will and S. Eckardt, "optiRiss - Simulation based Optimization and Risk Evaluation of Enhanced Geothermal Systems," in *Weimar Optimization and Stochastic Days*, Weimar, 2015.

[4] P. Brenda, "Geothermal Energy Resources," in *NARUC Winter Meeting*, February 2010.

[5] S. Nemat-Nasser, H. Abé and S. Hirakawa, "Hydraulic Fracturing and Geothermal Energy," in *Proceedings of First Japan-United States Joint Seminar on Hydraulic Fracturing and Geothermal Energy*, November 1982.

[6] S. Venkat, S. Eckardt and J. Will, "Simulation of Penny Shaped Fracture using homogenized continuum approach," 2016.

[7] Dynardo GmbH, Dynamic Software and Engineering GmbH, "multiPlas- Elastoplastic material models for Ansys," Dynardo GmbH, 2014. [Online]. Available: http://www.dynardo.de/en/software/multiplas.html.

[8] W. Wittke, Rock mechanics: theory and applications with case histories, The University of California: Springer, 1990, ISBN: 0387527192, 9780387527192.

[9] H. J. Kretzschmar and M. Rafiee, "MFRAC – Sensitivität, DBI – Zwischenbericht für OPTIRISS – Modellgruppe," DBI Freiberg, April 2013.

[10] J. Will and R. Schlegel, "Simulation of Hydraulic Fracturing of Jointed Rock," in *Proceedings of European Conference on Fracture*, Dresden, 2010.

[11] J. Will and T. Most, "Metamodell of Optimal Prognosis (MOP) - an Automatic Approach for User Friendly Parameter Optimization," in *Weimar Optimization and Stochastic Days*, Weimar, 2009.

[12] J. Will and T. Most, "Sensitivity analysis using the Metamodel of Optimal Prognosis," in *Weimar Optimization and Stochastic Days,* Weimar, 2011.

[13] F. Schlegel, "COMSOL BLOG," COMSOL GmbH, 30 May 2014. [Online]. Available: https://www.comsol.de/blogs/understanding-stabilization-methods/.

[14] S. Saeid and F. Barends, "An Extension of Lauwerier's Solution for Heat Flow in Saturated Porous Media," in *Proceedings of COMSOL Conference*, Milan, 2009.